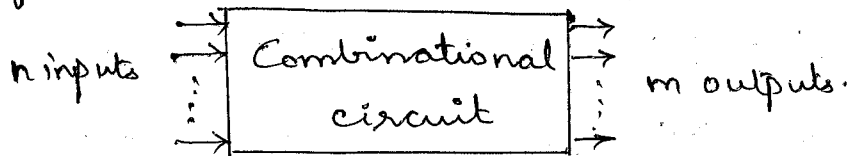


# UNIT-2

## COMBINATIONAL CIRCUITS.

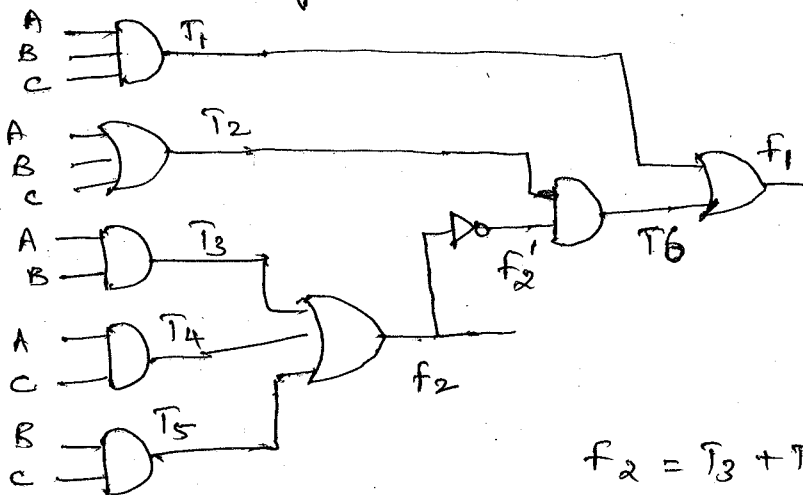
- Logic gates whose output at any time are determined from the present combination of input.
- performs an operation that can be specified logically by a set of boolean function
- No feedback.



### ANALYSIS PROCEDURE

1. Label all gate output that are a function of input variables with arbitrary symbols. Determine the boolean function for each gate output.
2. Label the gates that are a function of input variables and previously labelled gates with other arbitrary symbols. Find the boolean function for these gates.
3. Repeat the process until the outputs of the circuit are obtained.

1. Find boolean function and truth table for the circuit



$$T_1 = ABC$$

$$T_2 = A + B + C$$

$$T_3 = AB$$

$$T_4 = AC$$

$$T_5 = BC$$

$$f_2 = T_3 + T_4 + T_5$$

$$= AB + AC + BC \quad \text{--- (1)}$$

$$T_6 = T_2 f_2'$$

$$F_2' = (AB + AC + BC)'$$

$$= (AB)'(AC)'(BC)'$$

$$f_2' = (A'+B')(A'+c')(B'+c') \quad \text{--- (2)}$$

$$f_1 = T_1 + T_6$$

$$= ABC + T_2 f_2'$$

$$= ABC + [(A+B+c)(A'+B')(A'+c')(B'+c')]$$

$$= ABC + [(A+B+c)(A'+B'c')(B'+c')]$$

$$= ABC + [(A+B+c)(A'B'+A'c'+B'c'B'+B'c'c')]$$

$$= ABC + [A'B'+AA'c'+AB'c'+A'B'B'+A'c'B'+BB'c'+A'B'c'+A'c'c'+B'c'c']$$

$$f_1 = ABC + AB'c' + A'Bc' + A'B'c \quad \text{--- (3)}$$

$$f_2 = AB + AC + BC \quad \text{--- (4)}$$

TRUTH TABLE.

Let 3 variables be A, B, c.

Input			Output	
A	B	c	$f_1 = ABC + AB'c' + A'Bc' + A'B'c$	$f_2 = AB + AC + BC$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

DESIGN PROCEDURE

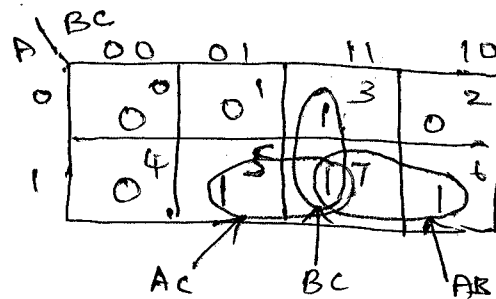
1. From the specification of circuit, determine the required number of inputs and outputs and assign a symbol to each.
2. Obtain the truth table that defines the required relationship between input and output.
3. Obtain the simplified boolean function using K-map.
4. Draw logic diagrams.

Q. Design a combinational circuits with three input variables that will produce a logic 1 output when more than one input variables are logic 1.

① Truth table

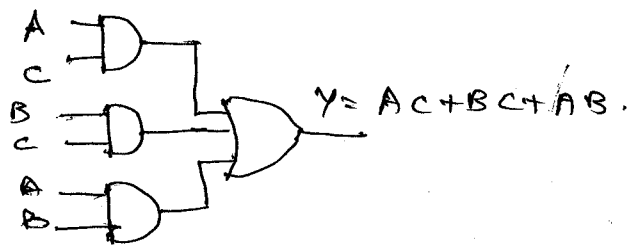
Input			Y
A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

② K-map



$Y = AC + BC + AB$

③ Logic diagram AND-OR



ADDER:

Addition consist of four possible elementary operations

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10_2$

full adder.

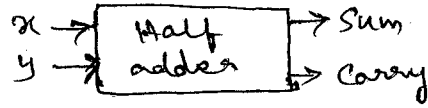
→ The combinational circuit that performs the addition of two bit is called half adder.

→ addition of three bit is a

HALF ADDER

• 2 bit addition, has 2 inputs  $x$  and  $y$  and two output sum and carry.

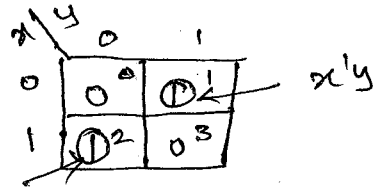
Augend  $\rightarrow x$   
Addend  $\rightarrow y$



Truth table

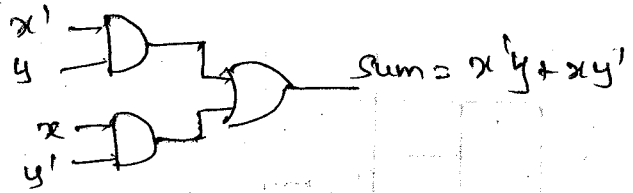
Input		Output	
$x$	$y$	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Sum K-map

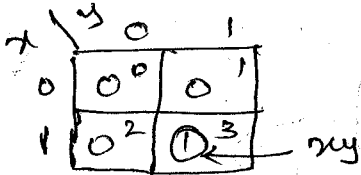


$xy'$  Sum =  $x'y + xy'$  (SOP)  
=  $x \oplus y$  (XOR)

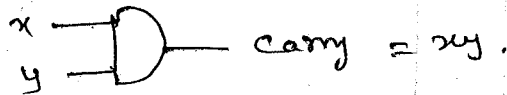
SOP implementation AND-OR



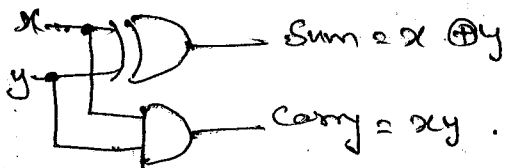
Carry K-map



Carry =  $xy$



EX-OR implementation



\* Carry output is one only when both inputs are one

\* Sum represents the least significant bit.

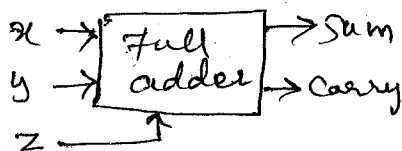
FULL ADDER

\* forms arithmetic sum of three bits.

\* consist of 3 inputs and 2 outputs

\*  $x, y$  input represent the two significant bits to be added and the third input  $z$ , represents the carry from the previous lower significant position

\* 2 output sum and carry.

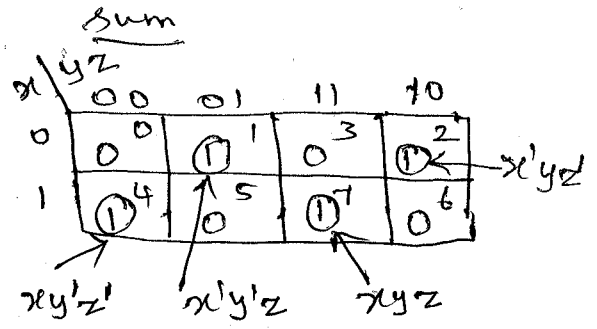


\* when all input bits are '0', the output is '0'.

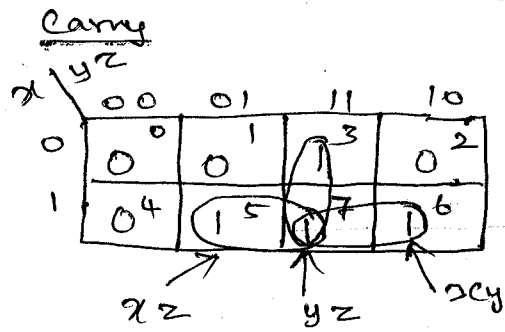
\* when only one input is equal to '1' then the sum = 1.

\* If 2 or 3 inputs are equal to 1, carry = 1.  
 Truth table

input			output	
x	y	z	Carry	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

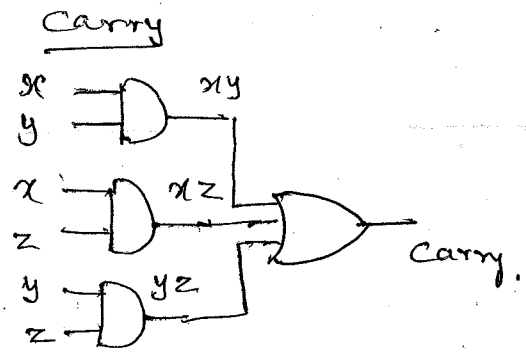
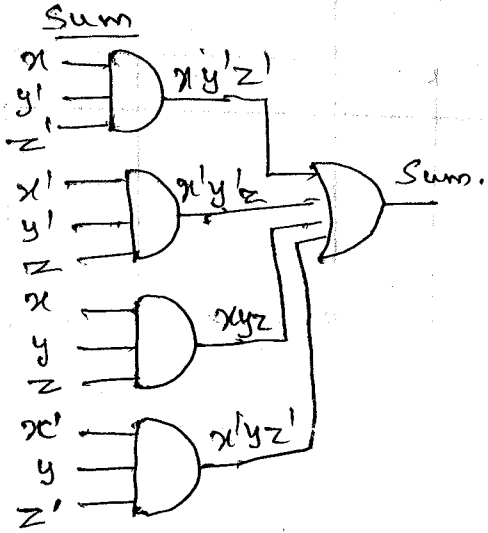


$$\text{Sum} = x'y'z + x'y'z' + xy'z + xy'z' \text{ (SOP)}$$



$$\text{Carry} = xz + yz + xy \text{ (SOP)}$$

SOP IMPLEMENTATION



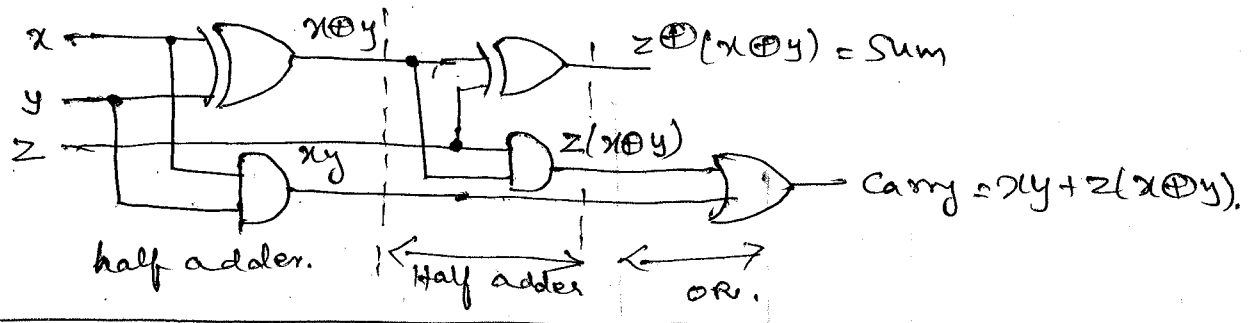
EX-OR implementation

EX-OR Implementation = Sum

$$\begin{aligned} \text{Sum} &= x'y'z + x'y'z' + xy'z + xy'z' \\ &= z(x'y' + xy') + z'(x'y + xy') \\ &= z(x \oplus y)' + z'(x \oplus y) \\ &= \underbrace{z}_{B} \oplus \underbrace{(x \oplus y)}_{A'} \\ &= \underbrace{z}_{B'} \oplus \underbrace{(x \oplus y)}_{A} \end{aligned}$$

Sum =  $z \oplus (x \oplus y)$

$$\begin{aligned} \text{Carry} &= xy + xz + yz \\ &= xy + z(x+y) \\ &= xy + z[x(y+y') + y(x+x')] \\ &= xy + z[xy + xy' + xy + xy'] \\ &= xy + xyz + z(x \oplus y) \\ &= xy(1+z) + z(x \oplus y) \\ \text{Carry} &= xy + z(x \oplus y) \end{aligned}$$

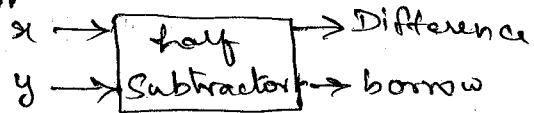


SUBTRACTOR

<u>Minuend</u>	-	<u>Subtrahend</u>	=	
0	-	0	=	0
0	-	1	=	1 with borrow 1
1	-	0	=	1
1	-	1	=	0

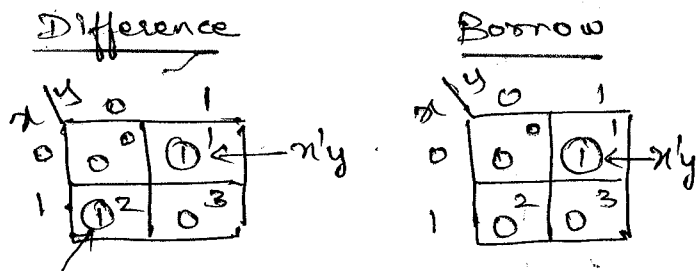
HALF SUBTRACTOR.

\* 2 bit subtraction, has 2 input x, y and 2 output difference and borrow.



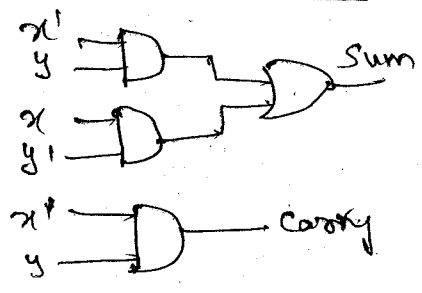
Truth table

input		output	
x	y	Difference	borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

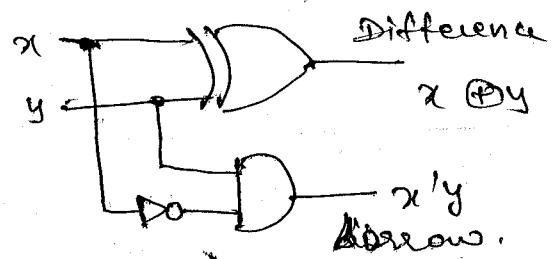


$\text{Difference} = x'y + xy'$  (SOP)  
 $= x \oplus y$  (XOR)  
 $\text{borrow} = x'y$

SOP Implementation

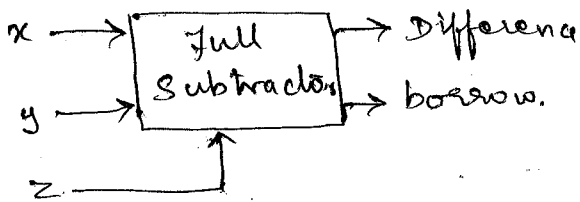


EX-OR implementation



FULL SUBTRACTOR

- \* It has 3 inputs and 2 outputs.
- \* x, y and z are inputs to be subtracted in which z represents borrow from the next stage.



Difference

x \ yz	00	01	11	10
0	0	1	0	1
1	1	0	1	0
	xyz	x'y'z	xyz	x'y'z'

$$\text{Difference} = x'y'z' + x'y'z + xyz + x'y'z'$$

Truth table

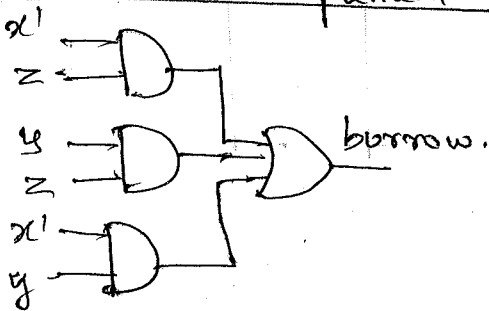
input			output	
x	y	z	Difference	borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Borrow

x \ yz	00	01	11	10
0	0	1	1	1
1	0	0	1	0
	x'z	yz	x'y	

$$\text{Borrow} = x'z + yz + x'y$$

Borrow - SOP implementation



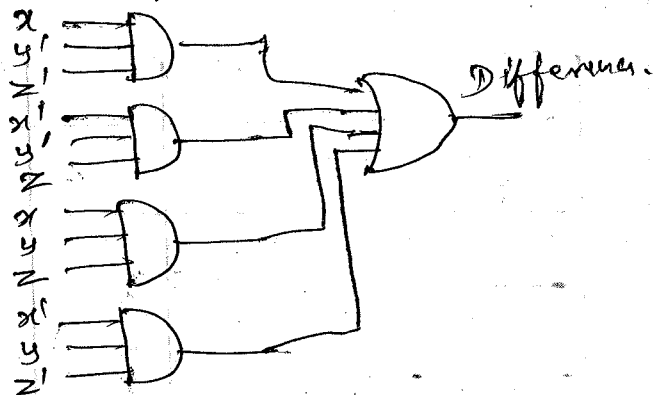
Borrow - EX-OR implementation

$$\text{Borrow} = x'z + yz + x'y$$

$$\begin{aligned} &= z(x' + y) + x'y \\ &= z[x'(y + y') + y(x + x')] + x'y \\ &= z(x'y + x'y' + xy + x'y') + x'y \\ &= z(x'y + (x \oplus y)') + x'y \\ &= x'y + z(x \oplus y)' \end{aligned}$$

SOP Implementation

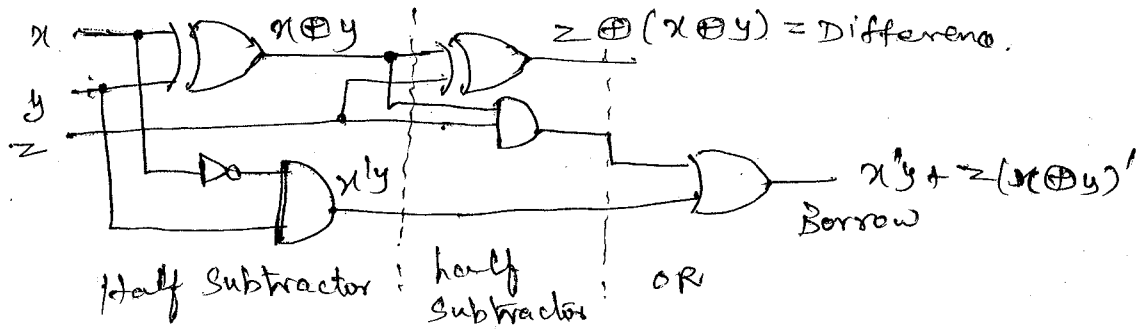
Difference



EX-OR implementation

$$\begin{aligned} \text{Difference} &= x'y'z' + x'y'z + xyz + x'y'z' \\ &= z'(x'y' + x'y) + z(xy + x'y) \\ &= z'(x \oplus y)' + z(x \oplus y) \\ &= z \oplus (x \oplus y) \end{aligned}$$

$$\begin{aligned} &= x'y(1+z) + z(x \oplus y)' \\ \text{Borrow} &= x'y + z(x \oplus y)' \end{aligned}$$



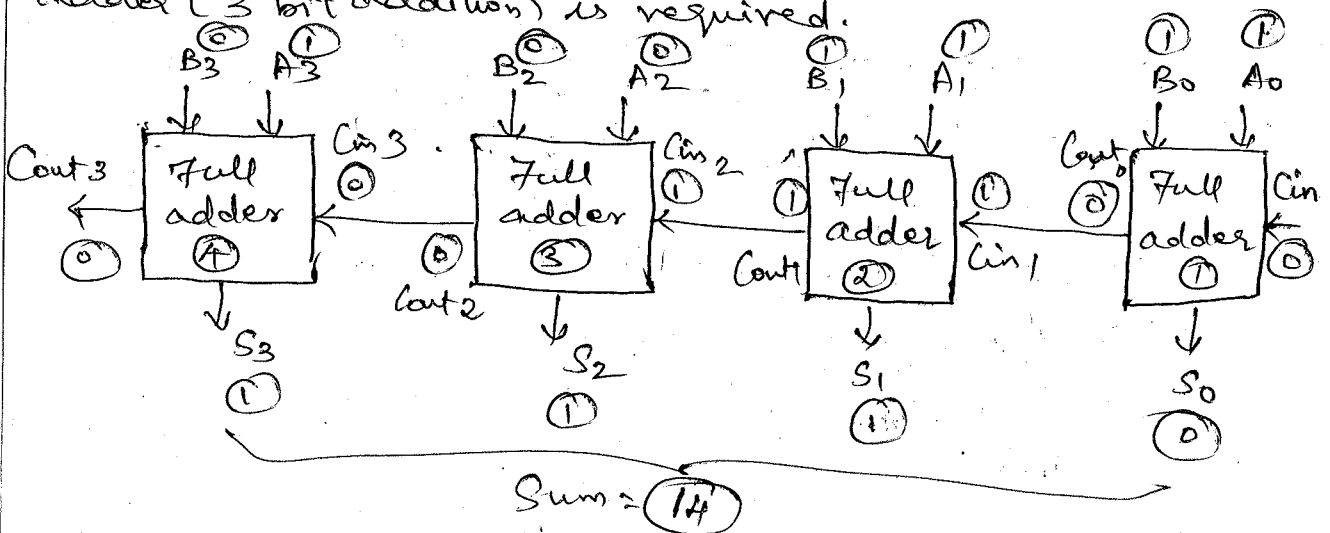
4 bit binary adder

- digital circuit that produces the arithmetic sum of two binary numbers.
- Constructed with full adder connected in cascade with output carry from each full adder connected as the input carry for the next full adder.

Let us consider two 4 bit number added taken as  $A_i$  and  $B_i$ .

Subscript $i$	3	2	1	0	
input carry $C_i$	0	1	1	0	
Augend $A_i$	1	0	1	1	(11)
Addend $B_i$	0	0	1	1	(3)
Sum $S_i$	1	1	1	0	(14) Ans.
Output carry $C_{i+1}$	0	0	1	1	

\* for adding two  $\rightarrow$  4 bit binary numbers we need a bit input carry  $\therefore$  for each bit addition one full adder (3 bit addition) is required.



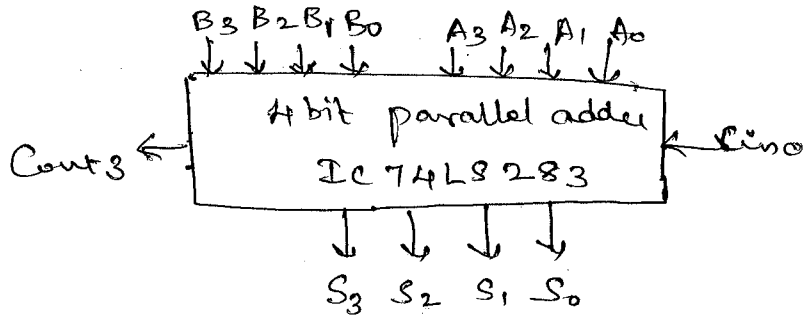
\* Always the input carry bit of LSB  $C_{in0}$  is equal to zero ( $C_{in0} = 0$ ) in full adder.

\* The output of full adder<sub>1</sub> has sum and carry, this carry is connected as the input carry  $C_{in1}$  of full adder<sub>2</sub> and continues....

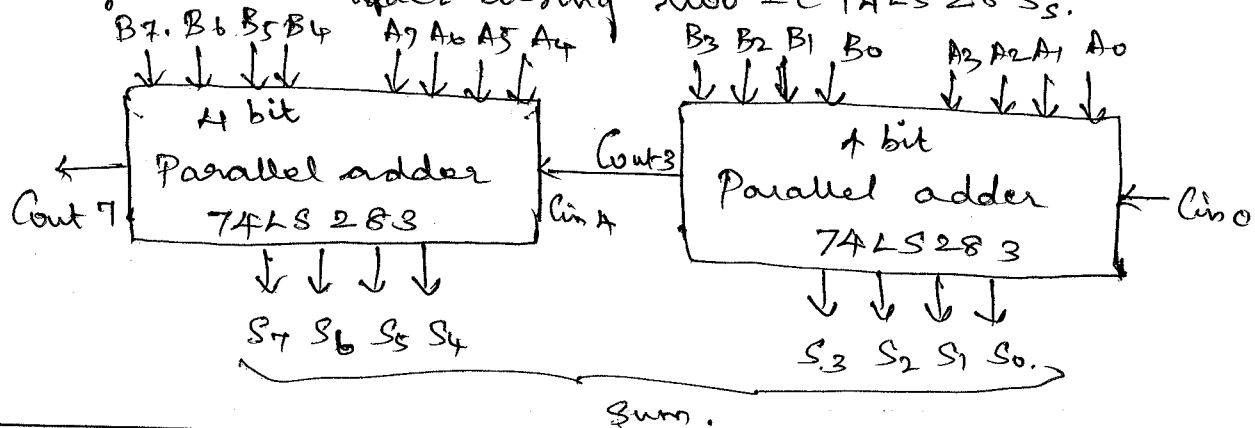
\* each full adder gives the one sum bit.

Design a 4 bit parallel adder using full-adder.

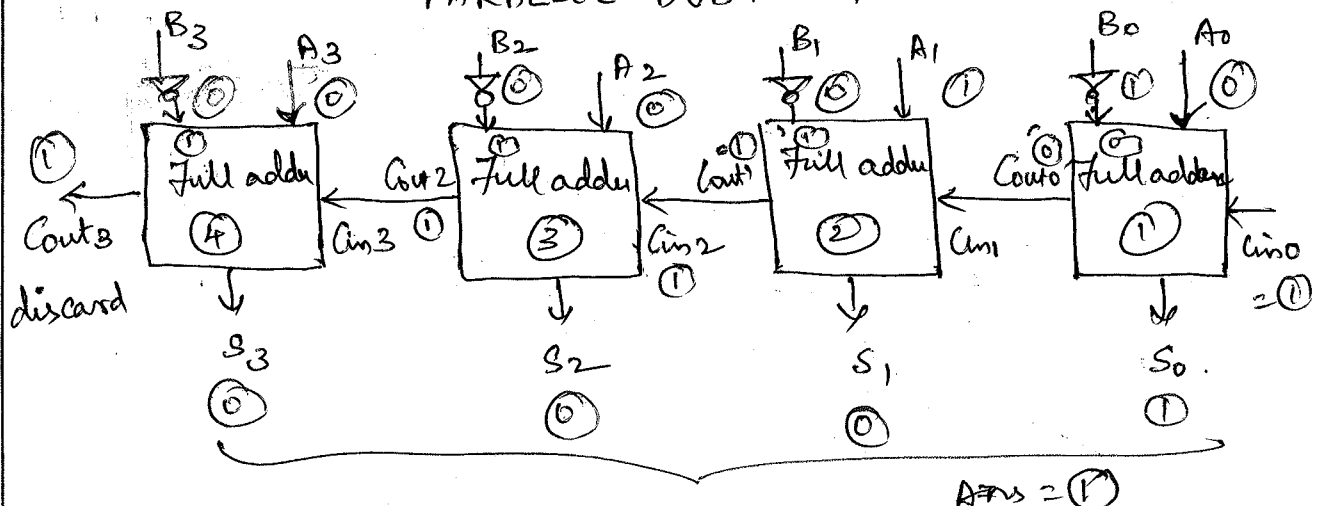
IC 74LS83 / 74LS283.



Design an 8 bit adder using two IC 74LS283s.



PARALLEL SUBTRACTOR.



\* Subtraction can be done by means of complements.

\* We know  $A-B$  means take 2's complement for B and add A.

\* 2's complement means take 1's complement + 1 to LSB.

\* for 1's complement use NOT gate and to add 1 to LSB is added to sum through input carry.

$$\begin{array}{r} A \quad B \\ 2-1 = 1 \rightarrow 0001 \\ \downarrow \quad \downarrow \\ 0010 \quad 0001 \end{array}$$

take 1's complement for  $B = 1110$

$$\begin{array}{r} +1 = \quad +1 \\ \hline 1111 \\ \hline \end{array}$$

add B to A

①  $A_3 \ A_2 \ A_1 \ A_0 = 2 = 0010$

$B_3 \ B_2 \ B_1 \ B_0 = 1 = 0001$

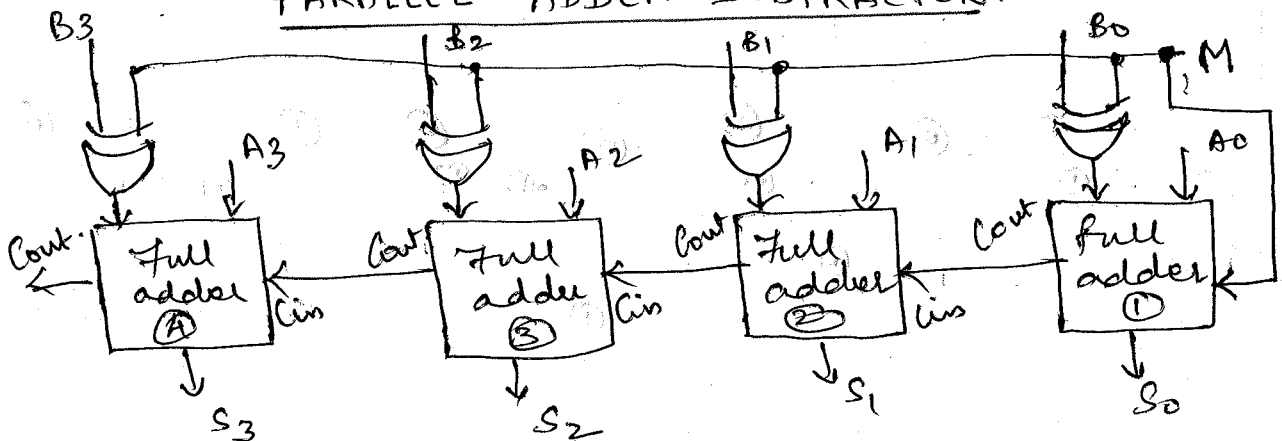
② Complement B (-) = 1110

③ Add A and B and } = 0010  
Carry = 1 } 1110

← Cin = 1

discard ← } 0001  
Carry. } ①

PARALLEL ADDER-SUBTRACTOR.

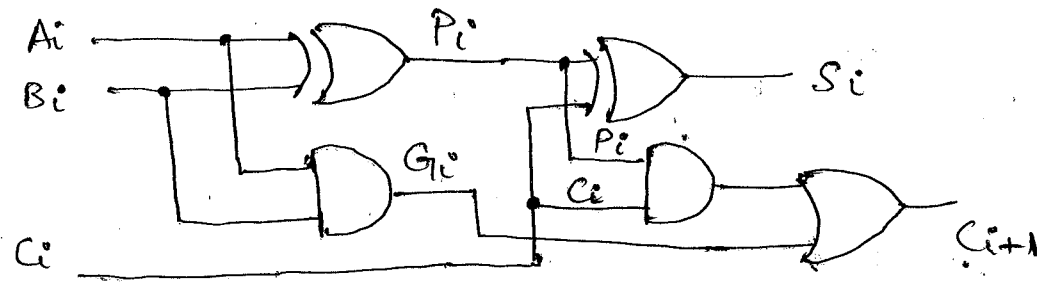


\* When  $M=0$ , the circuit is an adder, we have  $B \oplus 0 = B$ . The full adder receive the value of B, the input carry is '0' and the circuit performs  $A+B$ .

\* When  $M=1$ , the circuit becomes a subtractor, we have  $B \oplus 1 = \bar{B}$  and  $C_{in} = 1$ . The input to B are all complemented and 1 is added through the input carry. The circuit performs the operation  $A + 2$ 's complement of B (i.e.)  $A - B$ .

LOOK-AHEAD CARRY ADDER.

In the parallel adder the carry output of each full adder stage is connected to carry input of the next higher-order stage. The sum and carry outputs of any stage cannot be produced until the input carry occurs, this leads to time delay in addition process. This delay is known as carry propagation delay. If each full-adder is considered to have a propagation delay of 30ns, then  $S_3$  will not reach its correct value until 90ns after LSB carry is generated. Total time required to perform addition is  $90 + 30 = 120$ ns.



$P_i = A_i \oplus B_i$

$G_i = A_i B_i$

$S_i = P_i \oplus C_i$  — (1)

$C_{i+1} = G_i + P_i C_i$  — (2)

$i = 1$  in (2)

$C_2 = G_1 + P_1 C_1$  — (3)

$i = 2$  in (2)

$C_3 = G_2 + P_2 C_2$

Sub (3)  $C_3 = G_2 + P_2 (G_1 + P_1 C_1)$

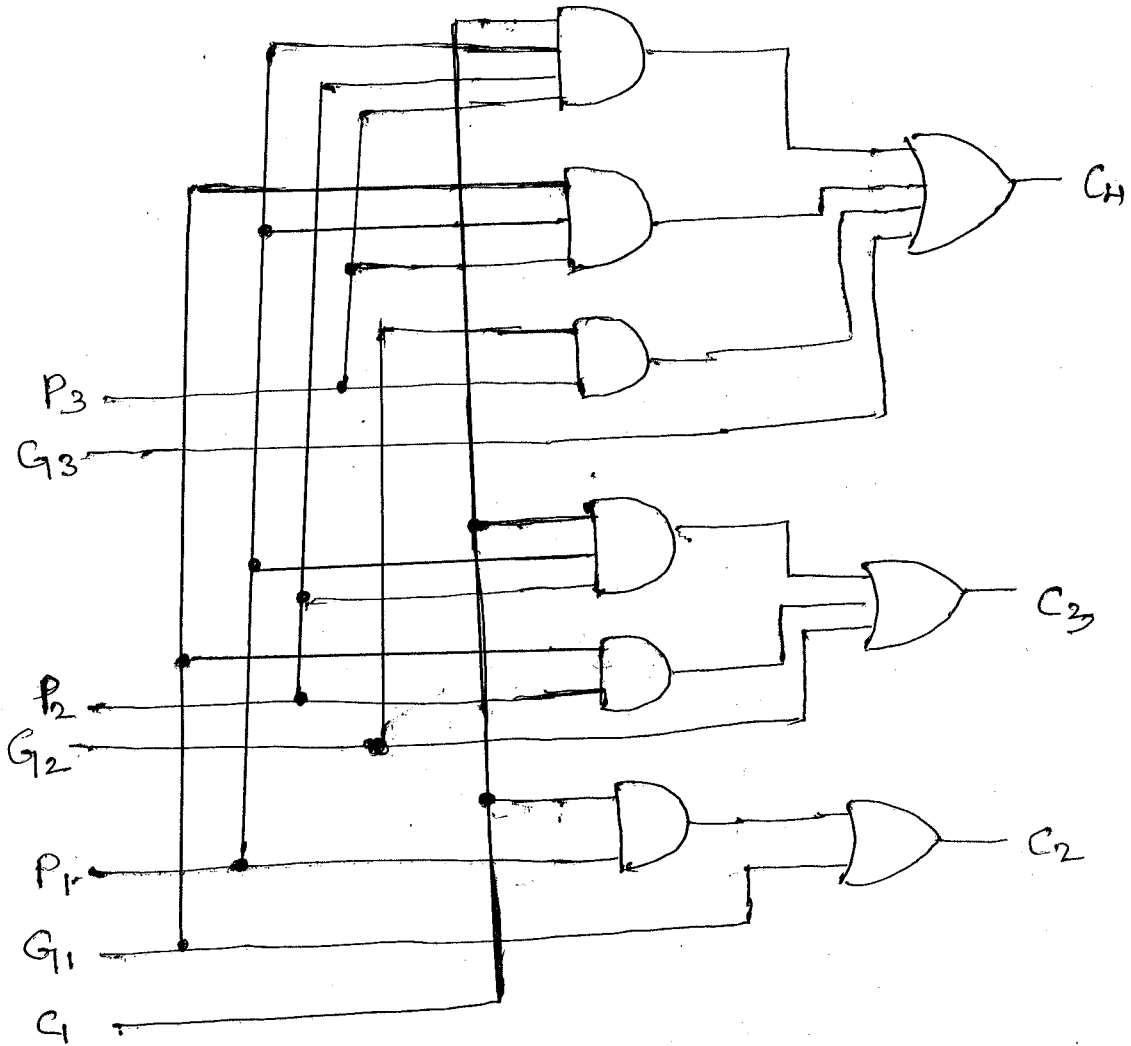
$$C_3 = G_2 + P_2 G_1 + P_2 P_1 C_1 \quad \text{--- (4)}$$

Sub  $i=3$  in (2)

$$C_4 = G_3 + P_3 C_3$$

$$\text{Sub (4)} = G_3 + P_3 [G_2 + P_2 G_1 + P_2 P_1 C_1]$$

$$C_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 C_1 \quad \text{--- (5)}$$

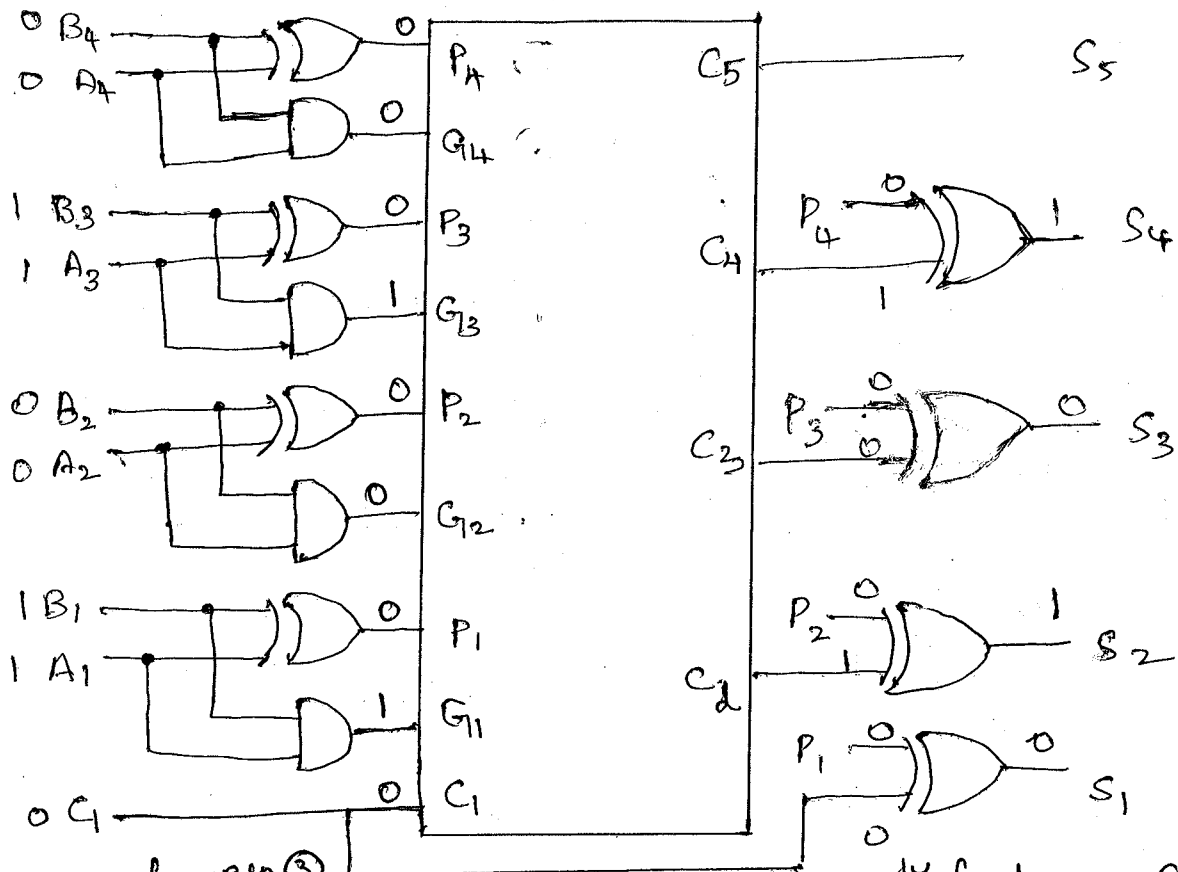


A bit parallel adder with look ahead carry generator.

	$C_4$	$C_3$	$C_2$	$C_1$
(5) A →	$A_4$	$A_3$	$A_2$	$A_1$
	0	1	0	1

(5) B →	$B_4$	$B_3$	$B_2$	$B_1$
	0	1	0	1

Sum =	$S_4$	$S_3$	$S_2$	$S_1$
	1	0	1	0



Note: from (3)  $C_2 = G_1 + P_1$ ,  $C_1 = 1 + 0 \cdot 0 = 1$   $\therefore$  find  $C_3, C_4$ .

\* EX-OR gate generates  $P_i$  and AND gate generates  $G_i$ .

\* Carriers are generated using look-ahead carry generator and applied as input to the second EX-OR gate.

\* The other input to EX-OR is  $P_i$ .

\* The second EX-OR gate generates sum output

\* Each output generated after a delay of two levels of gate.  $S_2$  through  $S_4$  has equal delay.

IC 74182 is a look-ahead carry generator

The 74182 carry look ahead generator accepts upto four pairs of active low carry propagate

( $\bar{P}_0, \bar{P}_1, \bar{P}_2, \bar{P}_3$ ) and carry generate high carries

( $C_{n+1}, C_{n+2}, C_{n+3}$ ) across four groups of binary adders. The 74182 also has active low carry

Propagate ( $\bar{P}$ ) and carry generate ( $\bar{G}$ ) outputs which may be used for further levels of look ahead.

The logic equations provided at the outputs of 74182

$$C_{n+x} = G_0 + P_0 C_n$$

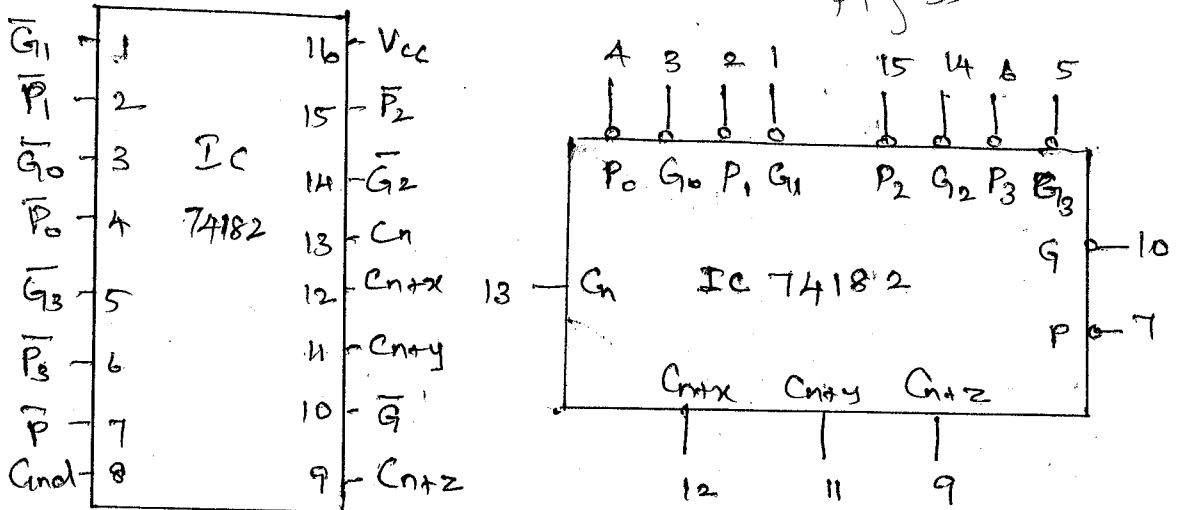
$$C_{n+y} = G_1 + P_1 G_0 + P_1 P_0 C_n$$

$$C_{n+z} = G_2 + P_2 G_1 + P_2 P_1 G_0$$

$$\bar{G} = \overline{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0}$$

$$\bar{P} = \overline{P_3 P_2 P_1 P_0}$$

Fig 35



1. Show the construction of 4 bit adder using IC 74182

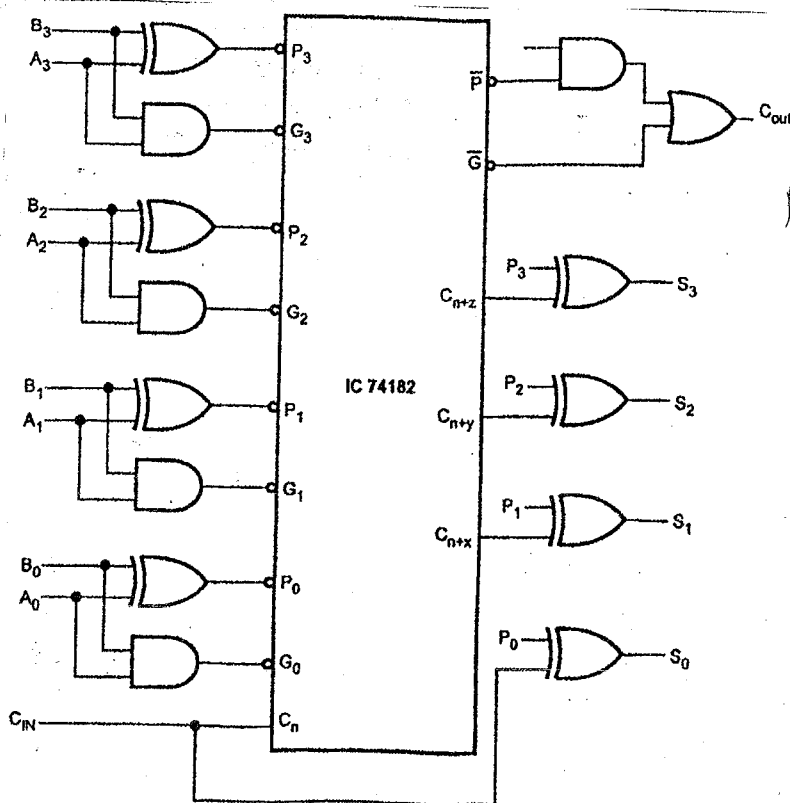
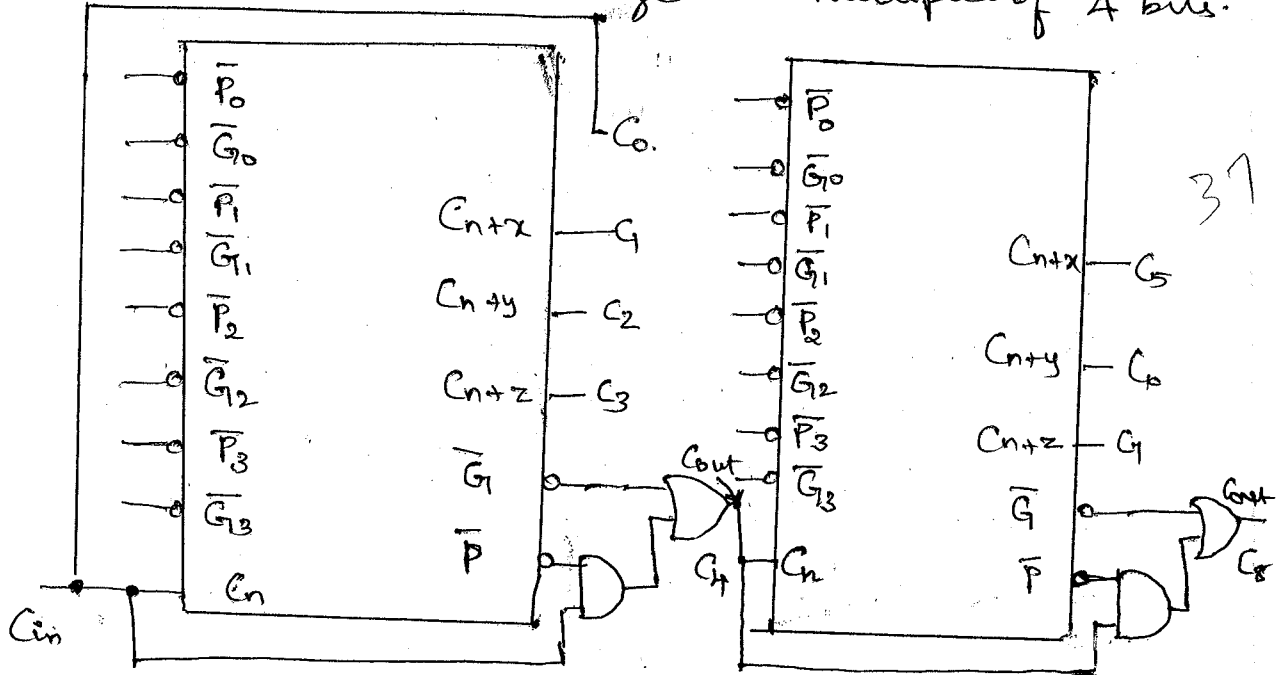


Fig 36

2. Construct the look ahead carry generator to accommodate higher word size.

look ahead carry generators can be cascaded to increase the word size in multiples of 4 bits.



CIRCUITS FOR ARITHMETIC OPERATION:

BCD ADDER.

- \* 4 bit binary adder for initial addition
- \* Logic circuit to detect  $sum > 9$ .
- \* One more 4 bit adder to add  $0110_2$  in the sum if sum is  $> 9$  or carry = 1.

Input				output
$S_3$	$S_2$	$S_1$	$S_0$	$\gamma$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

$> 9$

Input				output
$S_3$	$S_2$	$S_1$	$S_0$	$\gamma$
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

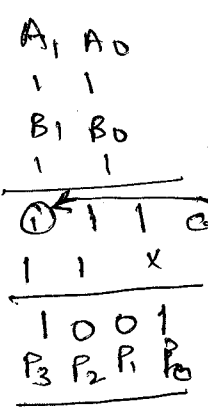
valid.  
No change  
add 0000

invalid  
add 0110



### BINARY MULTIPLIER 2 x 2 (Half adder)

0 x 0 = 0  
0 x 1 = 0  
1 x 0 = 0  
1 x 1 = 1

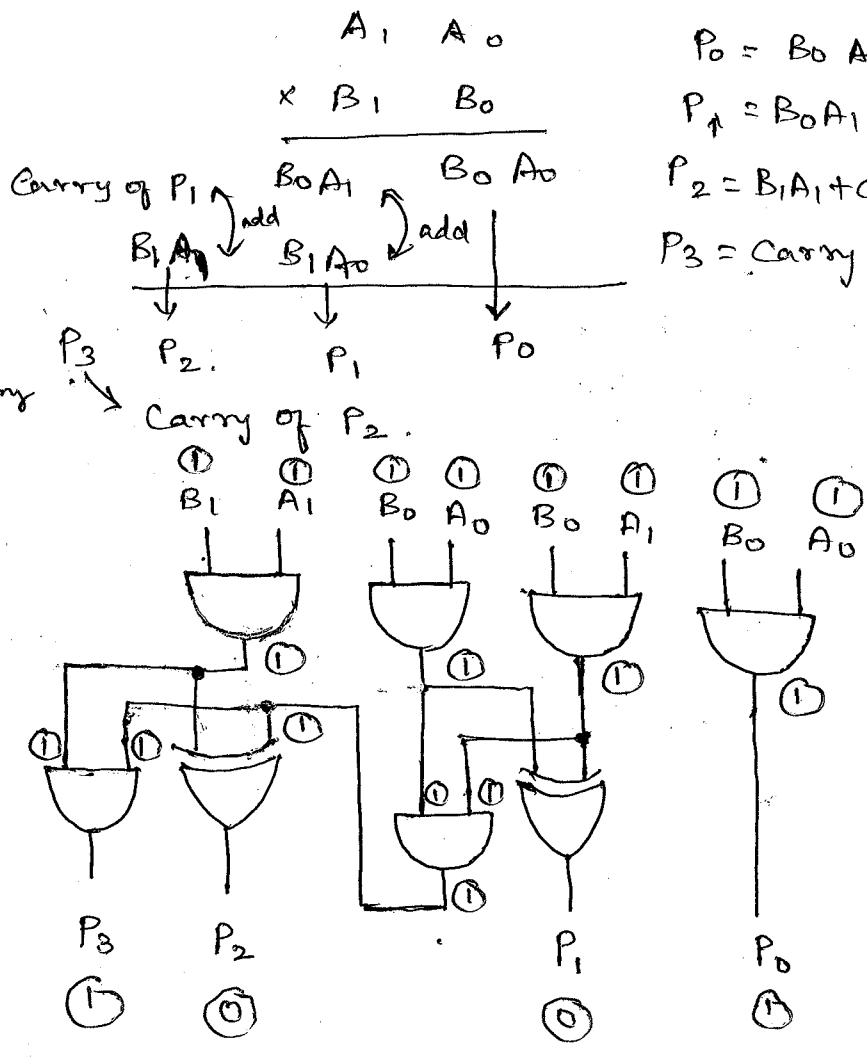


$$P_0 = B_0 A_0$$

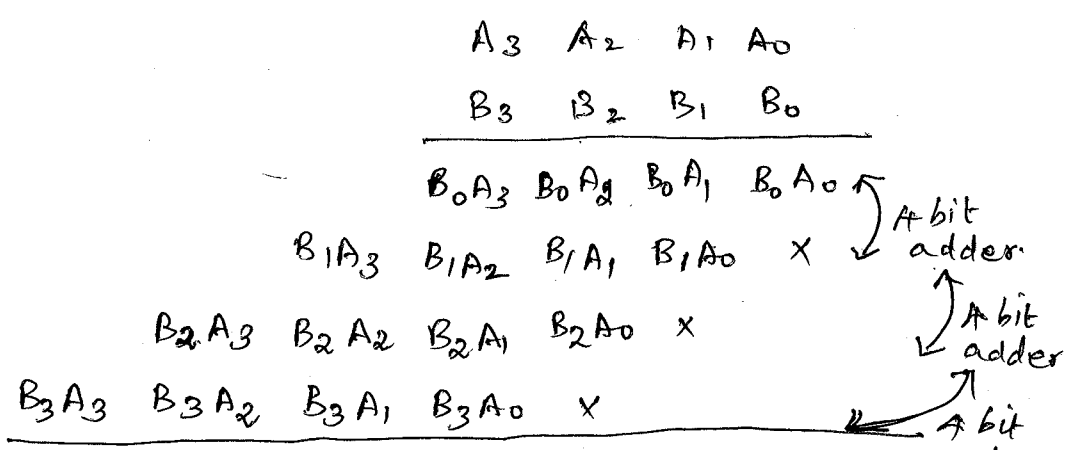
$$P_1 = B_0 A_1 + B_1 A_0$$

$$P_2 = B_1 A_1 + \text{Carry out of } P_1$$

$$P_3 = \text{Carry out of } P_2$$

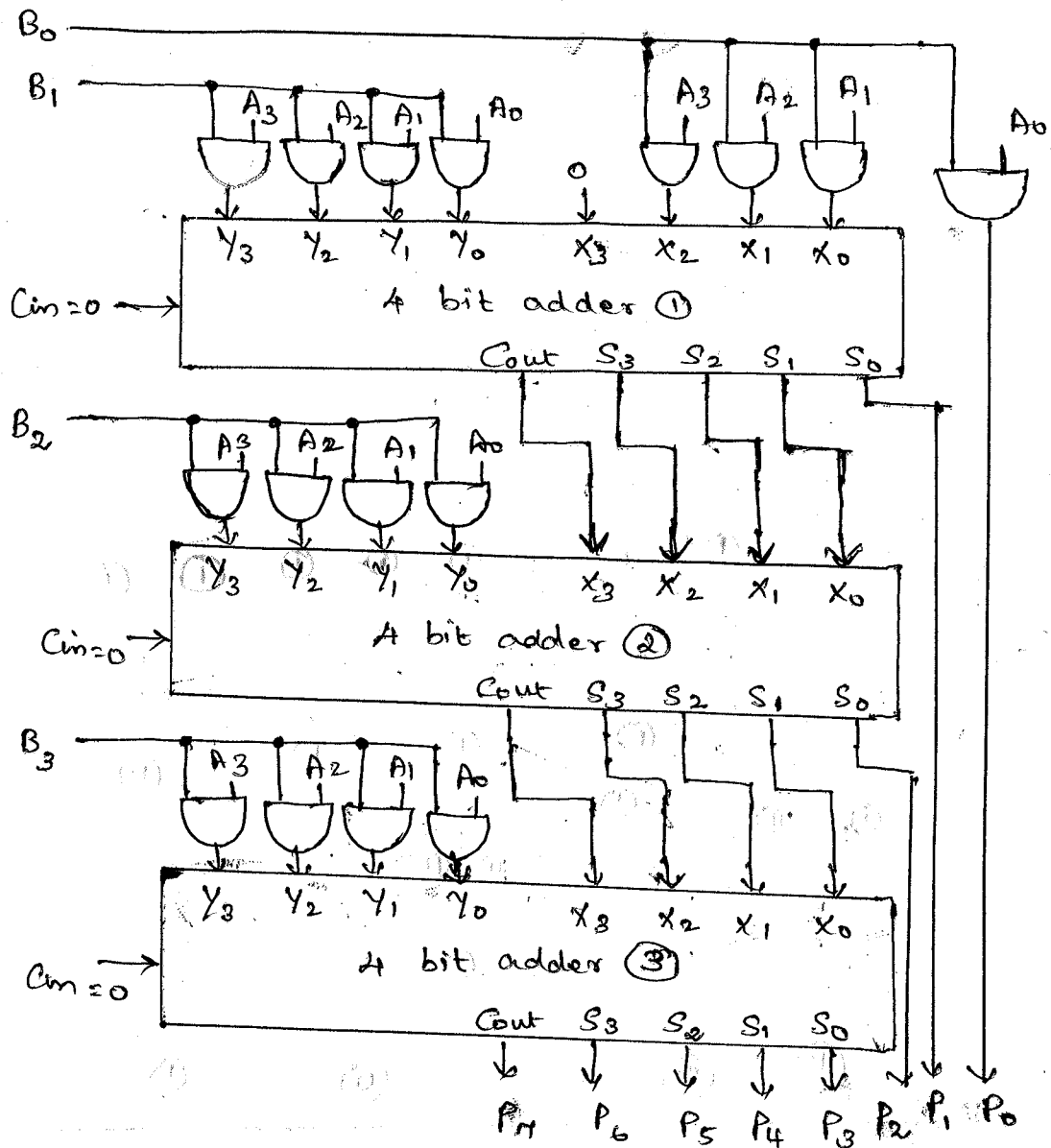


### 4 bit by 4 bit



P<sub>7</sub>      P<sub>6</sub>      P<sub>5</sub>      P<sub>4</sub>      P<sub>3</sub>      P<sub>2</sub>      P<sub>1</sub>      P<sub>0</sub>

↓  
Carry of P<sub>6</sub>.



$$P_0 = A_0 B_0$$

$$P_1 = B_0 A_1 + B_1 A_0$$

$$P_2 = B_0 A_2 + B_1 A_1 + B_2 A_0 + P_1 \text{ carry.}$$

$$P_3 = B_0 A_3 + B_1 A_2 + B_2 A_1 + B_3 A_0 + P_2 \text{ carry.}$$

$$P_4 = B_1 A_3 + B_2 A_2 + B_3 A_1 + P_3 \text{ carry.}$$

$$P_5 = B_2 A_3 + B_3 A_2 + P_4 \text{ carry.}$$

$$P_6 = B_3 A_3 + P_5 \text{ carry}$$

$$P_7 = P_6 \text{ carry.}$$

## CODE CONVERSION

There is a wide variety of binary codes used in digital systems. Some of these codes are binary-coded-decimal (BCD), Excess-3, Gray code and so on. Each code has different advantages and applications. Hence many times it is required to convert one code to another.

### 1. BINARY to BCD CONVERTER

Binary code				BCD code				
D	C	B	A	B <sub>4</sub>	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1	0
0	0	1	1	0	0	0	1	1
0	1	0	0	0	0	1	0	0
0	1	0	1	0	0	1	0	1
0	1	1	0	0	0	1	1	0
0	1	1	1	0	0	1	1	1
1	0	0	0	0	1	0	0	0
1	0	0	1	0	1	0	0	1
1	0	1	0	1	0	0	0	0
1	0	1	1	1	0	0	0	1
1	1	0	0	1	0	0	1	0
1	1	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	0
1	1	1	1	1	0	1	0	1

K-map

For B<sub>0</sub>

DC \ BA	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$B_0 = A$

For B<sub>1</sub>

DC \ BA	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	1	0	0
10	0	0	0	0

$B_1 = DC\bar{B} + \bar{D}B$

For B<sub>2</sub>

DC \ BA	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	1	1
10	0	0	0	0

$B_2 = \bar{D}C + CB$

For B<sub>3</sub>

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	1	0	0

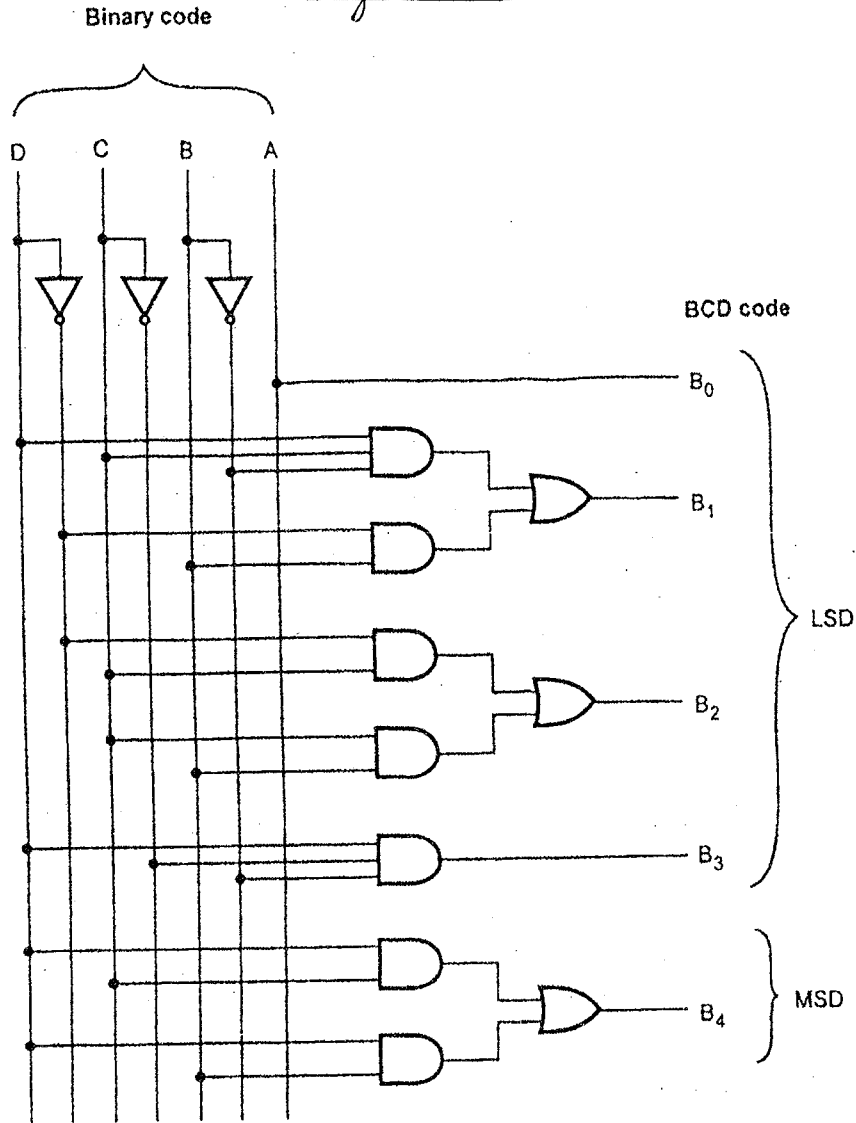
$B_3 = D\bar{C}\bar{B}$

For B<sub>4</sub>

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	1	1

$B_4 = DC + DB$

Logic diagram



Binary to BCD converter

BCD to BINARY CONVERTER

Truth table:

B <sub>4</sub>	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>	E	D	C	B	A
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1
0	0	0	1	0	0	0	0	1	0
0	0	0	1	1	0	0	0	1	1
0	0	1	0	0	0	0	1	0	0
0	0	1	0	1	0	0	1	0	1
0	0	1	1	0	0	0	1	1	0
0	0	1	1	1	0	0	1	1	1
0	1	0	0	0	0	1	0	0	0
0	1	0	0	1	0	1	0	0	1
1	0	0	0	0	0	1	0	1	0
1	0	0	0	1	0	1	0	1	1

1	0	0	1	0	0	1	1	0	0
1	0	0	1	1	0	1	1	0	1
1	0	1	0	0	0	1	1	1	0
1	0	1	0	1	0	1	1	1	1
1	0	1	1	0	1	0	0	0	0
1	0	1	1	1	1	0	0	0	1
1	1	0	0	0	1	0	0	1	0
1	1	0	0	1	1	0	0	1	1

Truth table for BCD to binary converter

K-map simplification

For A

		B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =0				B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =1	
		B <sub>3</sub> B <sub>2</sub>	00	01	11			10	B <sub>3</sub> B <sub>2</sub>	00	01
00	0	1	1	0	0	0	1	1	0	0	
01	0	1	1	0	0	0	1	1	0	0	
11	X	X	X	X	11	X	X	X	X	X	
10	0	1	X	X	10	0	1	X	X	X	

A = B<sub>0</sub>

For B

		B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =0				B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =1	
		B <sub>3</sub> B <sub>2</sub>	00	01	11			10	B <sub>3</sub> B <sub>2</sub>	00	01
00	0	0	1	1	00	1	1	0	0	0	
01	0	0	1	1	01	1	1	0	0	0	
11	X	X	X	X	11	X	X	X	X	X	
10	0	0	X	X	10	1	1	X	X	X	

$$B = B_1 \bar{B}_4 + \bar{B}_1 B_4$$

$$= B_1 \oplus B_4$$

For C

		B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =0				B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =1	
		B <sub>3</sub> B <sub>2</sub>	00	01	11			10	B <sub>3</sub> B <sub>2</sub>	00	01
00	0	0	0	0	00	0	0	1	1	0	
01	1	1	1	1	01	1	1	0	0	0	
11	X	X	X	X	11	X	X	X	X	X	
10	0	0	X	X	10	0	0	X	X	X	

$$C = \bar{B}_4 B_2 + B_2 \bar{B}_1$$

$$+ B_4 \bar{B}_2 B_1$$

For D

	B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =0	
B <sub>3</sub> B <sub>2</sub>	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	1	1	X	X

	B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =1	
B <sub>3</sub> B <sub>2</sub>	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

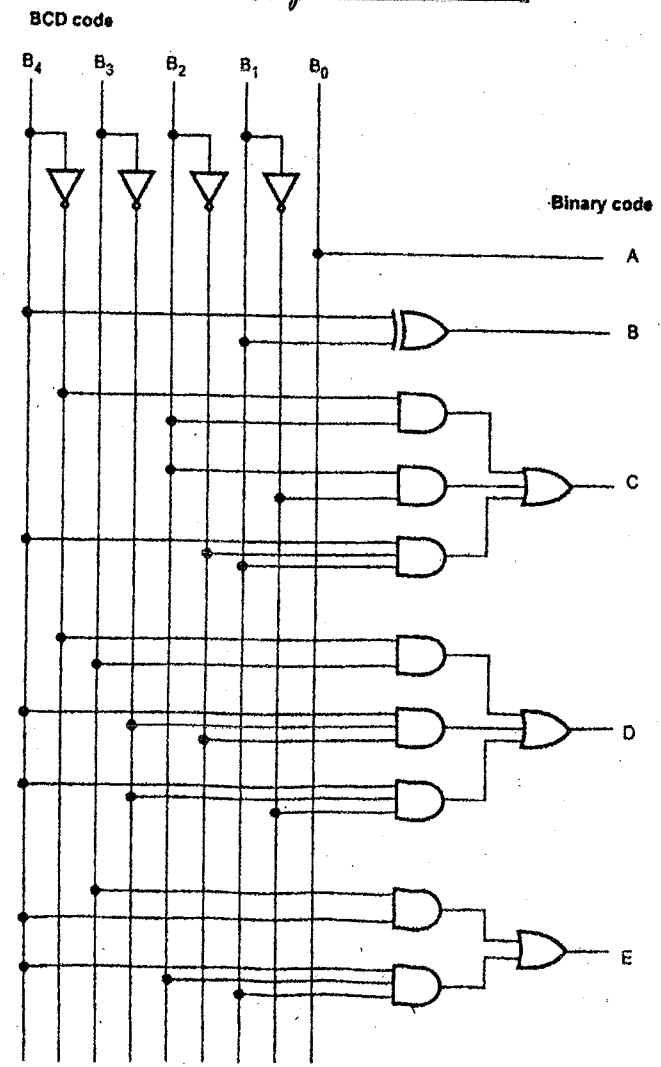
$$D = \bar{B}_4 B_3 + B_4 \bar{B}_3 \bar{B}_2 + B_4 \bar{B}_3 \bar{B}_1$$

For E

	B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =0	
B <sub>3</sub> B <sub>2</sub>	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	0	0	X	X

	B <sub>1</sub> B <sub>0</sub>		B <sub>4</sub> =1	
B <sub>3</sub> B <sub>2</sub>	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	X	X	X	X
10	1	1	X	X

Logic diagrams

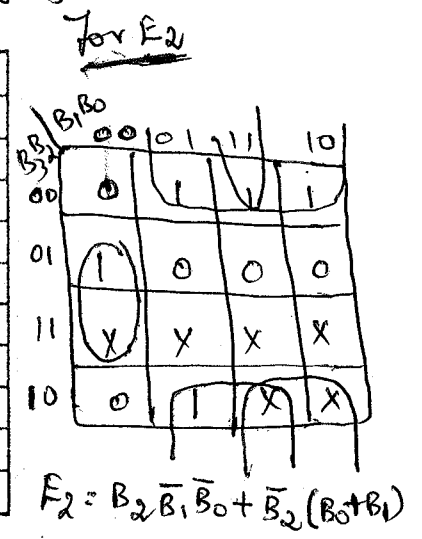


BCD to binary code converter

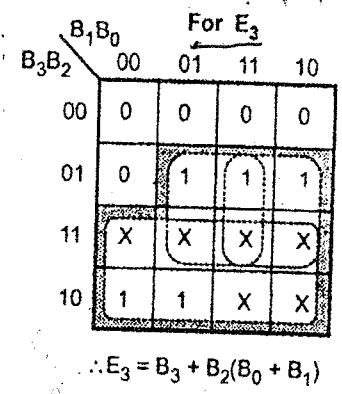
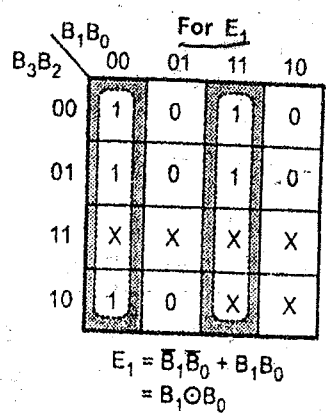
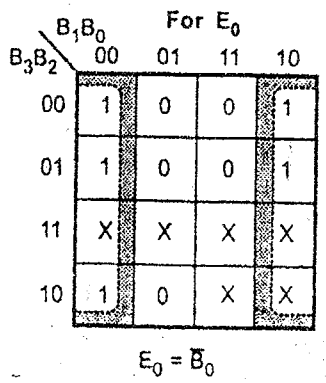
BCD to EXCESS-3.

- \* Excess-3 is modified form of BCD.
- \* Excess-3 code derived by adding 3 to BCD numbers.

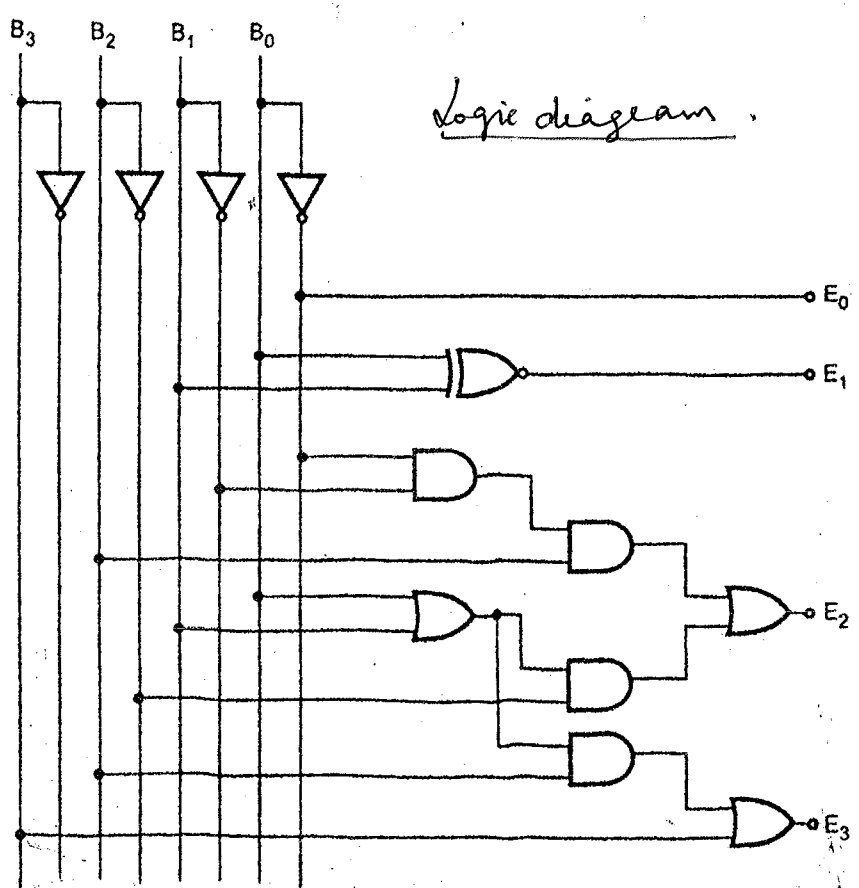
Decimal	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>	E <sub>3</sub>	E <sub>2</sub>	E <sub>1</sub>	E <sub>0</sub>
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0



K-map Simplification



BCD code



Excess-3 code

EXCESS-3 to CD Code Converter

Truth table

E <sub>3</sub>	E <sub>2</sub>	E <sub>1</sub>	E <sub>0</sub>	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1

K-map simplification

For B<sub>0</sub>

E <sub>3</sub> E <sub>2</sub> \ E <sub>1</sub> E <sub>0</sub>	00	01	11	10
00	X	X	0	X
01	1	0	0	1
11	1	X	X	X
10	1	0	0	1

$B_0 = \bar{E}_0$

For B<sub>1</sub>

E <sub>3</sub> E <sub>2</sub> \ E <sub>1</sub> E <sub>0</sub>	00	01	11	10
00	X	X	0	X
01	0	1	0	1
11	0	X	X	X
10	0	1	0	1

$B_1 = \bar{E}_1 E_0 + E_1 \bar{E}_0$   
 $= E_1 \oplus E_0$

For B<sub>2</sub>

E <sub>3</sub> E <sub>2</sub> \ E <sub>1</sub> E <sub>0</sub>	00	01	11	10
00	X	X	0	X
01	0	0	1	0
11	0	X	X	X
10	1	1	0	1

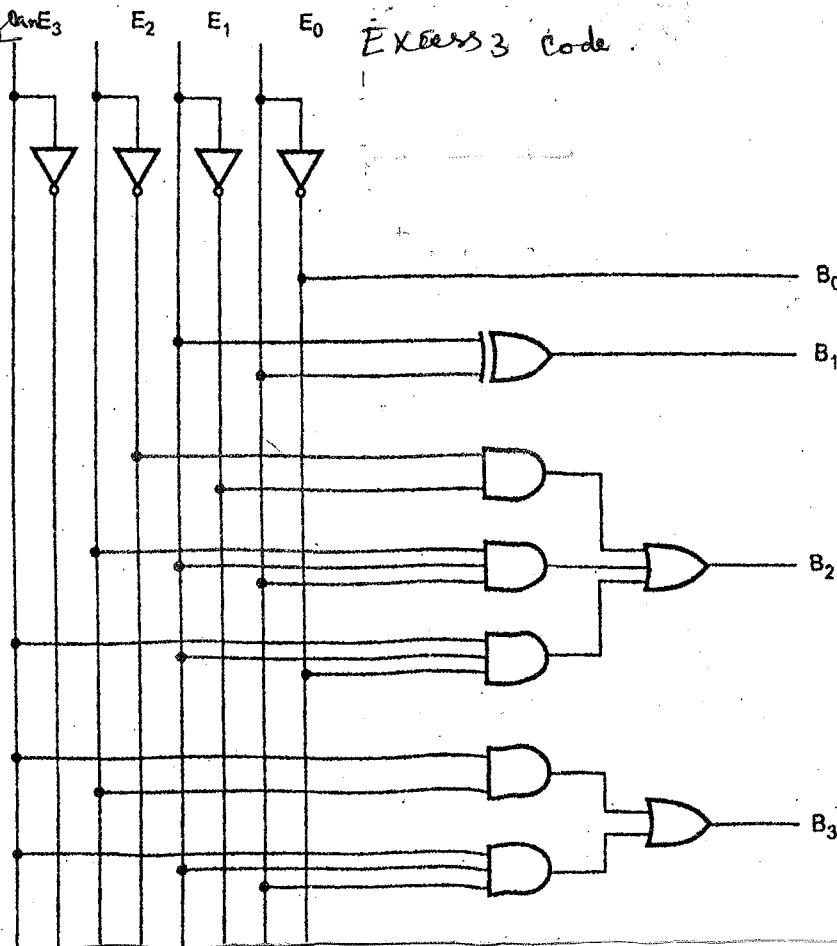
$B_2 = \bar{E}_2 \bar{E}_1 + E_2 E_1 E_0 + E_3 E_1 \bar{E}_0$

For B<sub>3</sub>

E <sub>3</sub> E <sub>2</sub> \ E <sub>1</sub> E <sub>0</sub>	00	01	11	10
00	X	X	0	X
01	0	0	0	0
11	1	X	X	X
10	0	0	1	0

$B_3 = E_3 E_1 + E_3 E_1 E_0$

Logic diagram



EXCESS-3 code

BCD code

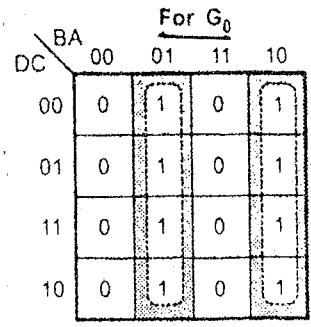
## BINARY to GRAY CODE CONVERTER

\* Gray code is often used in digital system has only one bit in numerical representation changes between successive numbers.

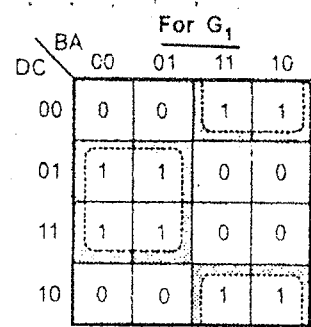
Truth table

Decimal	Binary code				Gray code			
	D	C	B	A	G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	1	0
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

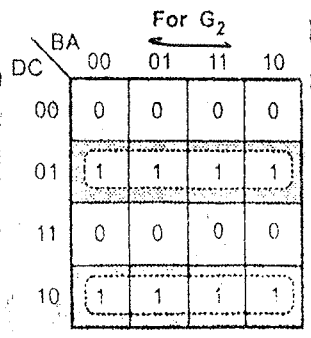
K-map Simplification



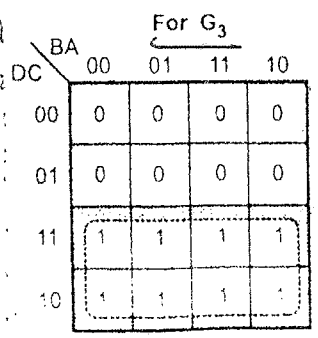
$$G_0 = \overline{B}A + B\overline{A} = B \oplus A$$



$$G_1 = C\overline{B} + \overline{C}B = C \oplus B$$

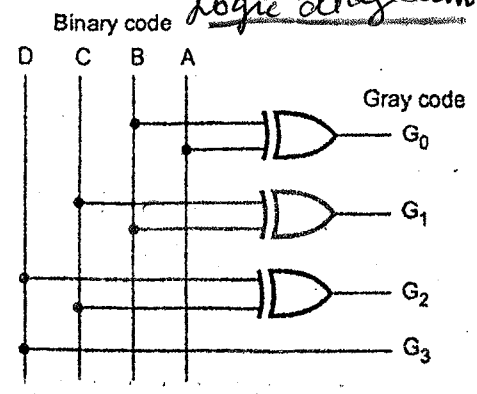


$$G_2 = \overline{D}C + D\overline{C} = D \oplus C$$



$$G_3 = D$$

Logic Diagram



## GRAY CODE to BINARY CODE CONVERTER

Truth table

Gray code				Binary code			
G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>	D	C	B	A
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1

0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

2

For B

	$G_1G_0$	00	01	11	10
$G_3G_2$	00	0	0	1	1
	01	1	1	0	0
	11	0	0	1	1
	10	1	1	0	0

$$\begin{aligned}
 B &= \bar{G}_3 \bar{G}_2 G_1 + \bar{G}_3 G_2 \bar{G}_1 + G_3 G_2 G_1 + G_3 \bar{G}_2 \bar{G}_1 \\
 &= G_1 (\bar{G}_3 \bar{G}_2 + G_3 G_2) + \bar{G}_1 (\bar{G}_3 G_2 + G_3 \bar{G}_2) \\
 &= G_1 (\bar{G}_3 \oplus G_2) + \bar{G}_1 (G_3 \oplus G_2) \\
 B &= G_1 \oplus (G_3 \oplus G_2)
 \end{aligned}$$

1. k-map Simplification

For A

	$G_1G_0$	00	01	11	10
$G_3G_2$	00	0	1	0	1
	01	1	0	1	0
	11	0	1	0	1
	10	1	0	1	0

$$\begin{aligned}
 A &= \bar{G}_3 \bar{G}_2 \bar{G}_1 G_0 + \bar{G}_3 \bar{G}_2 G_1 \bar{G}_0 + \bar{G}_3 G_2 \bar{G}_1 \bar{G}_0 + \bar{G}_3 G_2 G_1 G_0 + \\
 &\quad G_3 G_2 \bar{G}_1 G_0 + G_3 G_2 G_1 \bar{G}_0 + G_3 \bar{G}_2 \bar{G}_1 \bar{G}_0 + G_3 \bar{G}_2 G_1 G_0 \\
 &= \bar{G}_3 \bar{G}_2 (\bar{G}_1 G_0 + G_1 \bar{G}_0) + \bar{G}_3 G_2 (\bar{G}_1 \bar{G}_0 + G_1 G_0) + \\
 &\quad G_3 G_2 (\bar{G}_1 G_0 + G_1 \bar{G}_0) + G_3 \bar{G}_2 (\bar{G}_1 \bar{G}_0 + G_1 G_0) \\
 &= \bar{G}_3 \bar{G}_2 (G_1 \oplus G_0) + \bar{G}_3 G_2 (\bar{G}_1 \oplus \bar{G}_0) + G_3 G_2 (G_1 \oplus G_0) \\
 &\quad G_3 \bar{G}_2 (\bar{G}_1 \oplus \bar{G}_0) \\
 &= \{(G_1 \oplus G_0) [\bar{G}_3 \bar{G}_2 + G_3 G_2]\} + \{(\bar{G}_1 \oplus \bar{G}_0) (\bar{G}_3 \bar{G}_2 + G_3 \bar{G}_2)\} \\
 &= (G_1 \oplus G_0) (\bar{G}_3 \oplus G_2) + (\bar{G}_1 \oplus \bar{G}_0) (G_3 \oplus G_2) \\
 A &= (G_1 \oplus G_0) \oplus (G_3 \oplus G_2)
 \end{aligned}$$

Eq 10

For C

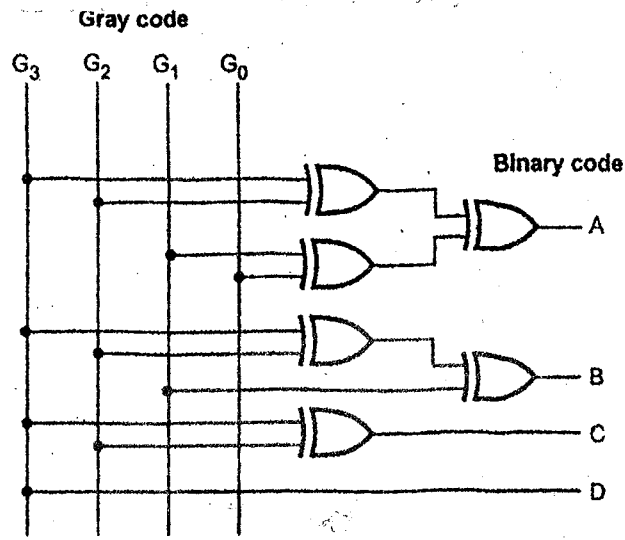
	$G_1G_0$	00	01	11	10
$G_3G_2$	00	0	0	0	0
	01	1	1	1	1
	11	0	0	0	0
	10	1	1	1	1

$$\begin{aligned}
 C &= \bar{G}_3 G_2 + G_3 \bar{G}_2 \\
 C &= G_3 \oplus G_2
 \end{aligned}$$

For D

	$G_1G_0$	00	01	11	10
$G_3G_2$	00	0	0	0	0
	01	0	0	0	0
	11	1	1	1	1
	10	1	1	1	1

$$D = G_3$$



BCD to GRAY CODE CONVERTER.

BCD code				Gray code			
B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>	G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1

For G<sub>0</sub>

B <sub>3</sub> B <sub>2</sub> \ B <sub>1</sub> B <sub>0</sub>	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	X	X	X	X
10	0	1	X	X

$$G_0 = \bar{B}_1 B_0 + B_1 \bar{B}_0$$

$$= B_1 \oplus B_0$$

For G<sub>1</sub>

B <sub>3</sub> B <sub>2</sub> \ B <sub>1</sub> B <sub>0</sub>	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

$$G_1 = B_2 \bar{B}_1 + \bar{B}_2 B_1$$

$$= B_2 \oplus B_1$$

For G<sub>2</sub>

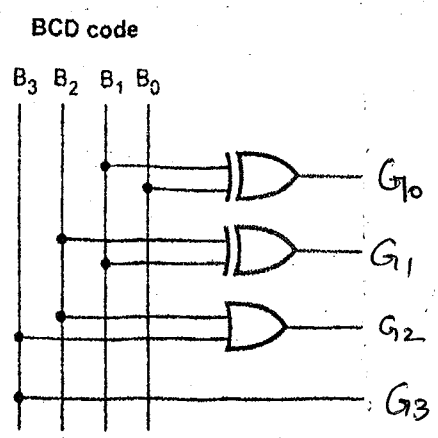
B <sub>3</sub> B <sub>2</sub> \ B <sub>1</sub> B <sub>0</sub>	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	X	X	X	X
10	1	1	X	X

$$G_2 = B_2 + B_3$$

For G<sub>3</sub>

B <sub>3</sub> B <sub>2</sub> \ B <sub>1</sub> B <sub>0</sub>	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	1	1	X	X

$$G_3 = B_3$$



GRAY TO BCD CODE CONVERTER.

Gray code				BCD code			
G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>	D	C	B	A
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	x	x	x	x
1	1	1	0	x	x	x	x
1	0	1	0	x	x	x	x
1	0	1	1	x	x	x	x
1	0	0	1	x	x	x	x
1	0	0	0	x	x	x	x

For A

G <sub>3</sub> G <sub>2</sub> \ G <sub>1</sub> G <sub>0</sub>	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

$$\begin{aligned}
 A &= \bar{G}_3 \bar{G}_2 \bar{G}_1 G_0 + \bar{G}_2 G_1 \bar{G}_0 + G_2 \bar{G}_1 \bar{G}_0 \\
 &\quad + G_2 G_1 G_0 + G_3 \bar{G}_0 \\
 &= G_2 (\bar{G}_1 \bar{G}_0 + G_1 G_0) + \bar{G}_0 (G_2 G_1 + G_3) \\
 &\quad + G_3 \bar{G}_2 \bar{G}_1 G_0 \\
 &= G_2 (G_1 \oplus G_0) + \bar{G}_0 (G_2 G_1 + G_3) + G_3 \bar{G}_2 \bar{G}_1 G_0
 \end{aligned}$$

For B

G <sub>3</sub> G <sub>2</sub> \ G <sub>1</sub> G <sub>0</sub>	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	X	X	X	X
10	1	1	X	X

$$B = G_2 \bar{G}_1 + G_3 + \bar{G}_2 G_1$$

For C

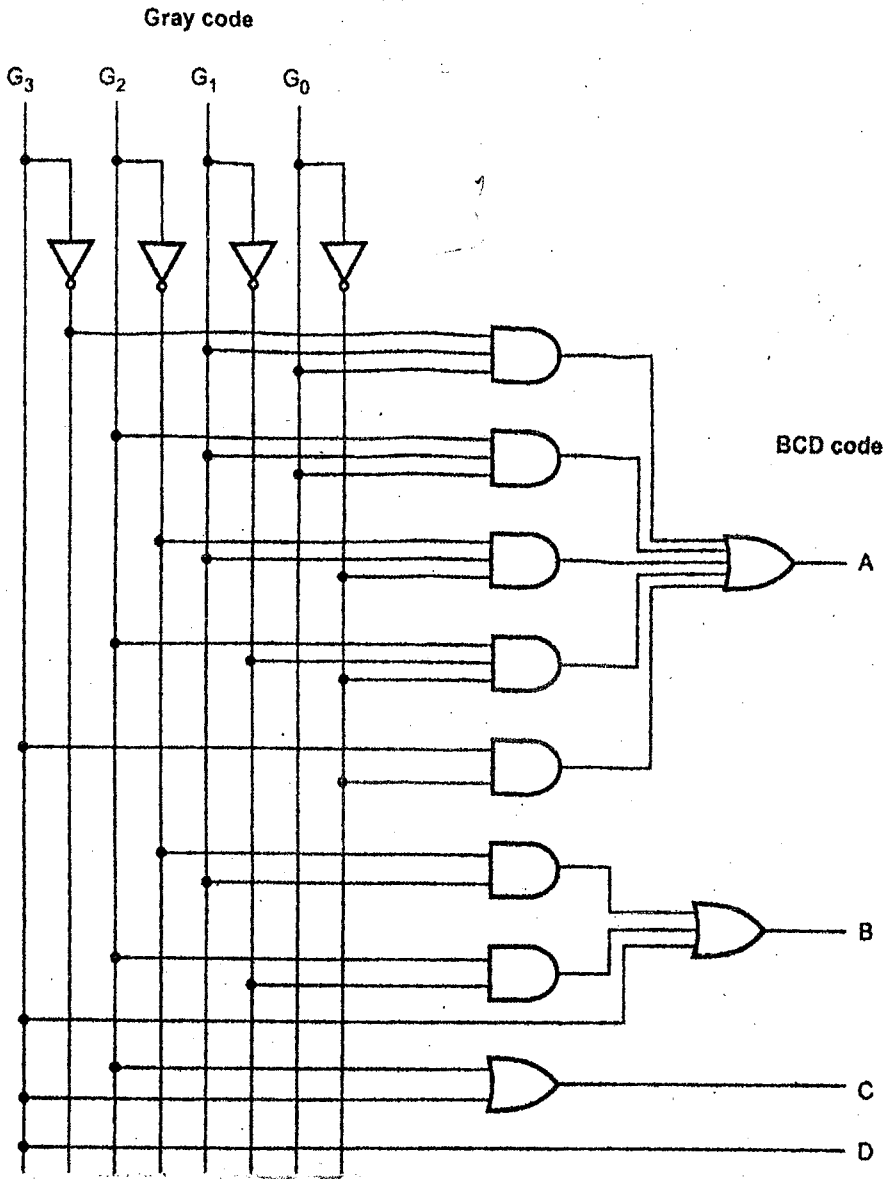
G <sub>3</sub> G <sub>2</sub> \ G <sub>1</sub> G <sub>0</sub>	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	X	X	X	X
10	1	1	X	X

$$C = G_2 + G_3$$

For D

G <sub>3</sub> G <sub>2</sub> \ G <sub>1</sub> G <sub>0</sub>	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	1	1	X	X

$$D = G_3$$



4 bit Gray code to 2 digit BCD.

Gray code				BCD code				
G <sub>3</sub>	G <sub>2</sub>	G <sub>1</sub>	G <sub>0</sub>	E	D	C	B	A
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	1
0	0	1	1	0	0	0	1	0
0	0	1	0	0	0	0	1	1
0	1	1	0	0	0	1	0	0
0	1	1	1	0	0	1	0	1
0	1	0	1	0	0	1	1	0
0	1	0	0	0	0	1	1	1
1	1	0	0	0	1	0	0	0
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	0	0
1	1	1	0	1	0	0	0	1
1	0	1	0	1	0	0	1	0
1	0	1	1	1	0	0	1	1
1	0	0	1	1	0	1	0	0
1	0	0	0	1	0	1	0	1

For A

	$G_1 G_0$			
	00	01	11	10
$G_3 G_2$	00	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$$\begin{aligned}
 A &= \bar{G}_3 \bar{G}_2 \bar{G}_1 G_0 + \bar{G}_3 \bar{G}_2 G_1 \bar{G}_0 + \bar{G}_3 G_2 \bar{G}_1 G_0 \\
 &+ \bar{G}_3 G_2 G_1 G_0 + G_3 \bar{G}_2 \bar{G}_1 G_0 + G_3 \bar{G}_2 G_1 \bar{G}_0 \\
 &+ G_3 G_2 \bar{G}_1 G_0 + G_3 G_2 G_1 \bar{G}_0 + G_3 \bar{G}_2 \bar{G}_1 \bar{G}_0 \\
 &+ G_3 \bar{G}_2 G_1 G_0 \\
 &= \bar{G}_3 \bar{G}_2 (\bar{G}_1 G_0 + G_1 \bar{G}_0) + \bar{G}_3 G_2 (\bar{G}_1 G_0 + G_1 G_0) \\
 &+ G_3 \bar{G}_2 (\bar{G}_1 G_0 + G_1 \bar{G}_0) + G_3 G_2 (\bar{G}_1 \bar{G}_0 + G_1 G_0)
 \end{aligned}$$

$$\begin{aligned}
 A &= \bar{G}_3 \bar{G}_2 (G_1 \oplus G_0) + \bar{G}_3 G_2 (\bar{G}_1 \oplus G_0) + G_3 \bar{G}_2 (G_1 \oplus G_0) + \\
 &G_3 G_2 (\bar{G}_1 \oplus G_0) \\
 &= \{(G_1 \oplus G_0) (\bar{G}_3 \bar{G}_2 + G_3 G_2)\} + \{(\bar{G}_1 \oplus G_0) (\bar{G}_3 G_2 + G_3 \bar{G}_2)\} \\
 &= (G_1 \oplus G_0) (\bar{G}_3 \oplus G_2) + (\bar{G}_1 \oplus G_0) (G_3 \oplus G_2)
 \end{aligned}$$

$$A = (G_1 \oplus G_0) \oplus (G_3 \oplus G_2)$$

For B

	$G_1 G_0$			
	00	01	11	10
$G_3 G_2$	00	0	1	1
01	1	1	0	0
11	0	0	0	0
10	0	0	1	1

$$B = \bar{G}_2 G_1 + \bar{G}_3 G_2 \bar{G}_1$$

For C

	$G_1 G_0$			
	00	01	11	10
$G_3 G_2$	00	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

$$C = \bar{G}_3 G_2 + G_3 \bar{G}_2 \bar{G}_1$$

For D

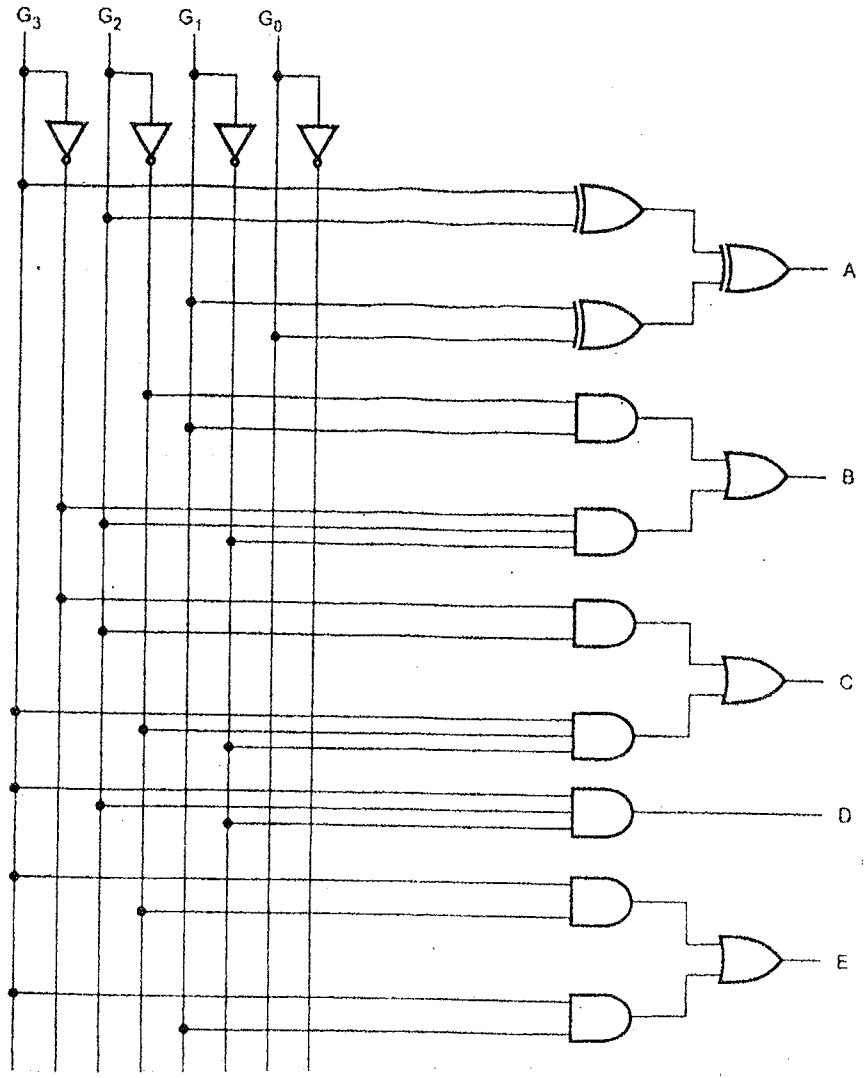
	$G_1 G_0$			
	00	01	11	10
$G_3 G_2$	00	0	0	0
01	0	0	0	0
11	1	1	0	0
10	0	0	0	0

$$D = G_3 G_2 \bar{G}_1$$

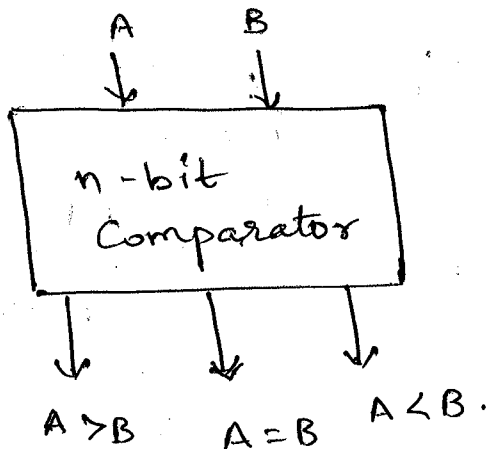
For E

	$G_1 G_0$			
	00	01	11	10
$G_3 G_2$	00	0	0	0
01	0	0	0	0
11	0	0	1	1
10	1	1	1	1

$$E = G_3 \bar{G}_2 + G_3 G_1$$



The circuit that compare relative magnitude of two binary numbers is called magnitude comparator circuit.



Block diagram of Magnitude Comparator.

1 bit Magnitude Comparator

It compare one bit binary numbers.

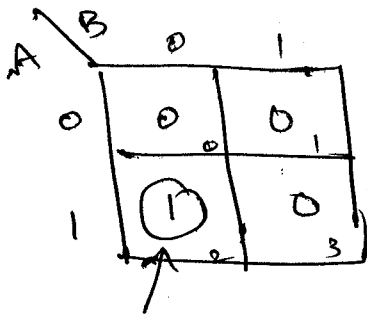
Inputs A & B.  
 A = 0      B = 0  
       1      1

Truth table

input		output Y		
A	B	$Y_{A > B}$	$Y_{A = B}$	$Y_{A < B}$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

K-map.

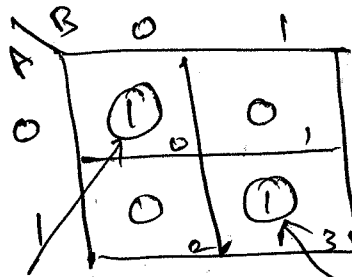
$Y(A > B)$



$A \bar{B}$

$Y(A > B) = A \bar{B}$

$Y(A = B)$

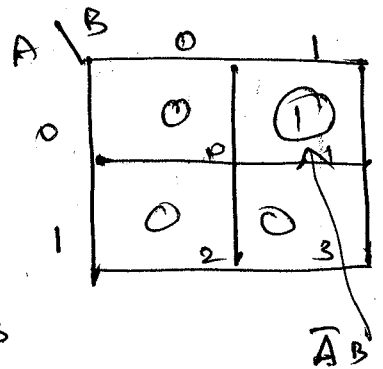


$\bar{A} \bar{B}$

$AB$

$Y_{A=B} = AB + \bar{A} \bar{B}$   
 $= A \oplus B$

$Y(A < B)$

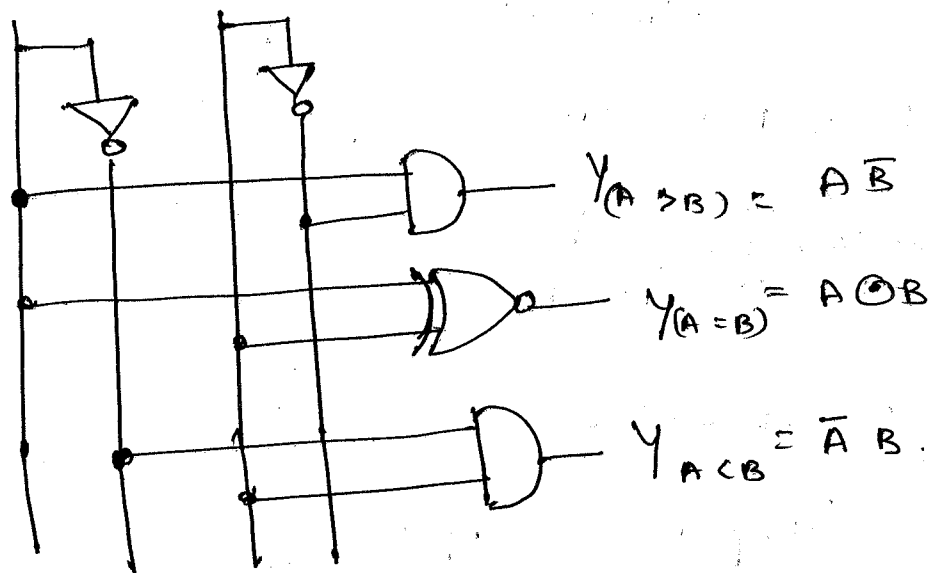


$\bar{A} B$

$Y_{A < B} = \bar{A} B$

Logic diagram.

Inputs



2 bit Magnitude Comparator:

It compares 2 bit numbers. Let the inputs be A and B. Where,

$A = A_1 A_0$

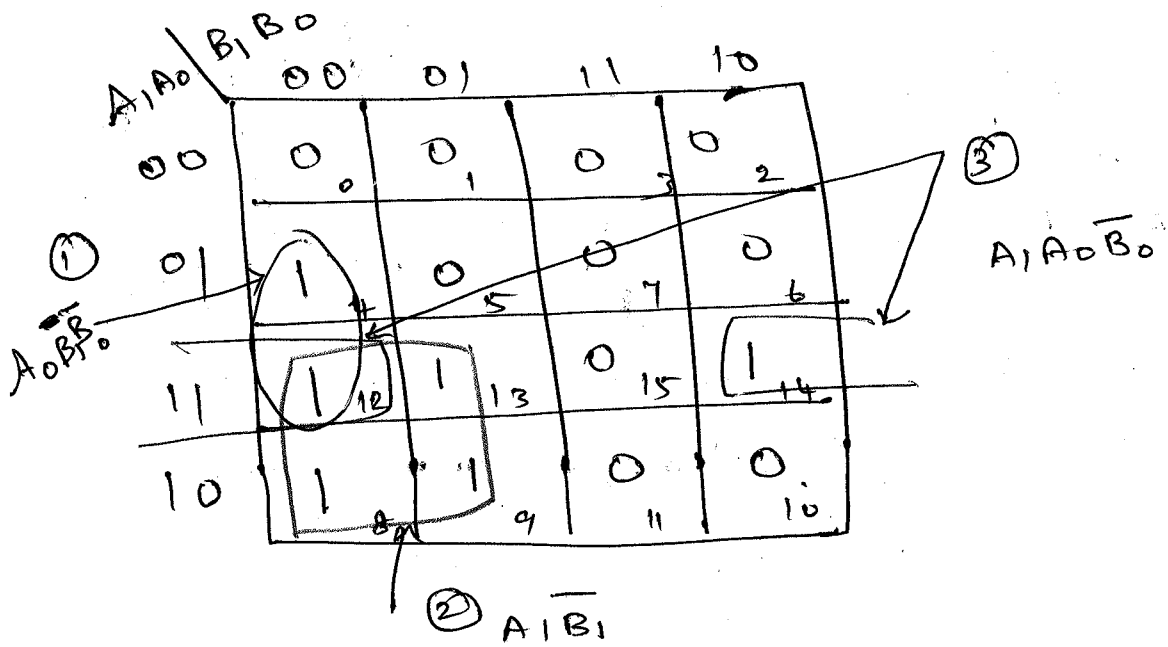
$B = B_1 B_0$

} each 2 bit binary number.

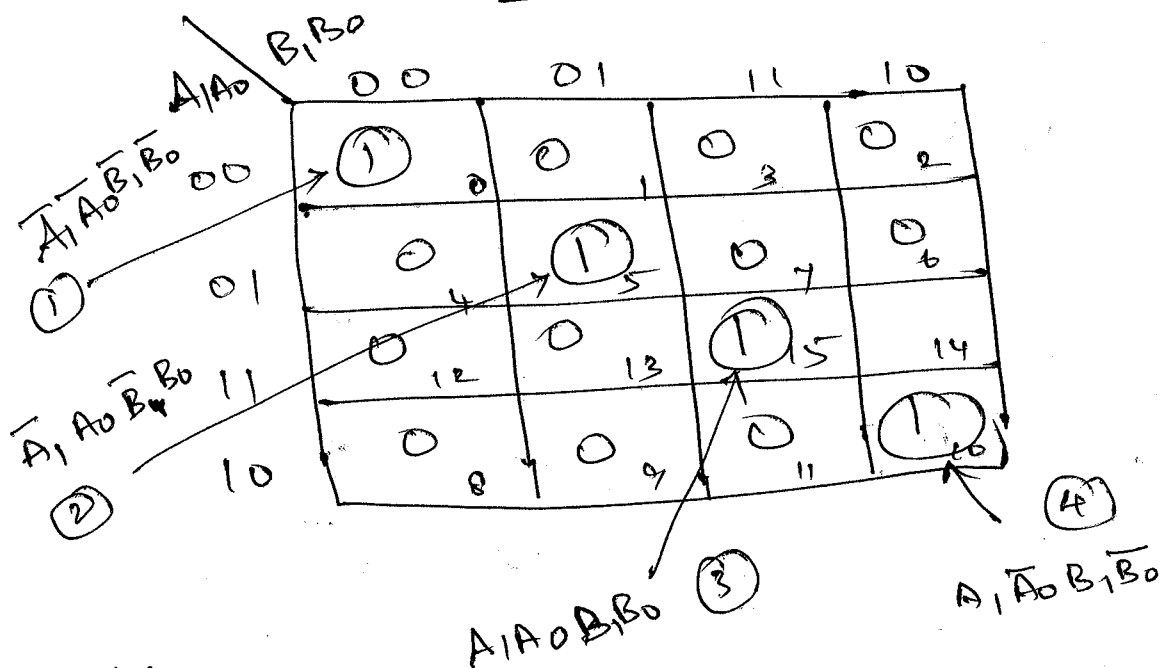
Truth table

inputs				Output		
A		B		Y		
A <sub>1</sub>	A <sub>0</sub>	B <sub>1</sub>	B <sub>0</sub>	Y <sub>A&gt;B</sub>	Y <sub>A=B</sub>	Y <sub>A&lt;B</sub>
0	0	0	0	0	1	0
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	0	1
0	1	0	0	1	0	0
		0	1	0	1	0
		1	0	0	0	1
		1	1	0	0	1
1	0	0	0	1	0	0
		0	1	1	0	0
		1	0	0	1	0
		1	1	0	0	1
1	1	0	0	1	0	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	0	1	0

K map

 $Y_{A > B}$ 

$$Y_{(A > B)} = A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{B}_1 + A_1 A_0 \bar{B}_0$$

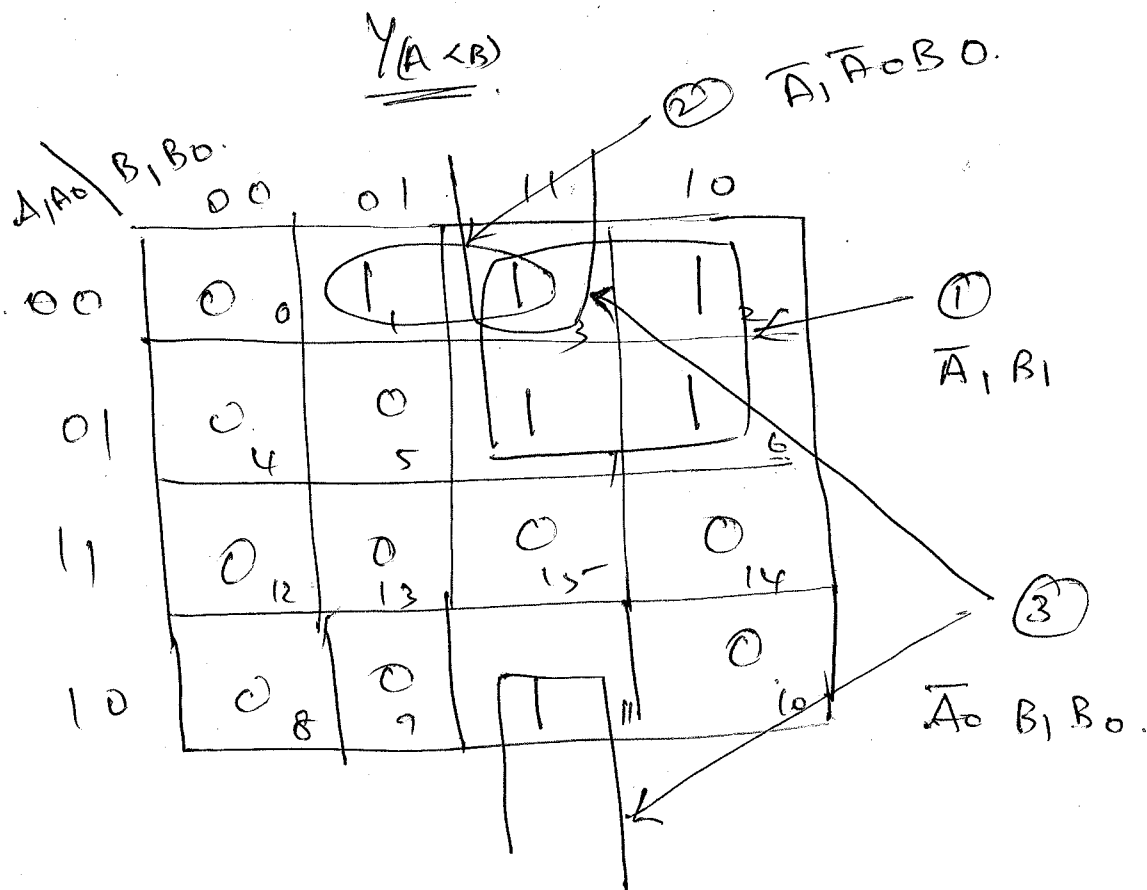
 $Y_{A = B}$ 

$$\begin{aligned}
 Y_{(A = B)} &= \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + \\
 &A_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0 \\
 &= \bar{A}_1 \bar{B}_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0)
 \end{aligned}$$

$$= \bar{A}_1 \bar{B}_1 (A_0 \odot B_0) + A_1 B_1 (A_0 \odot B_0)$$

$$= (A_0 \odot B_0) (\bar{A}_1 \bar{B}_1 + A_1 B_1)$$

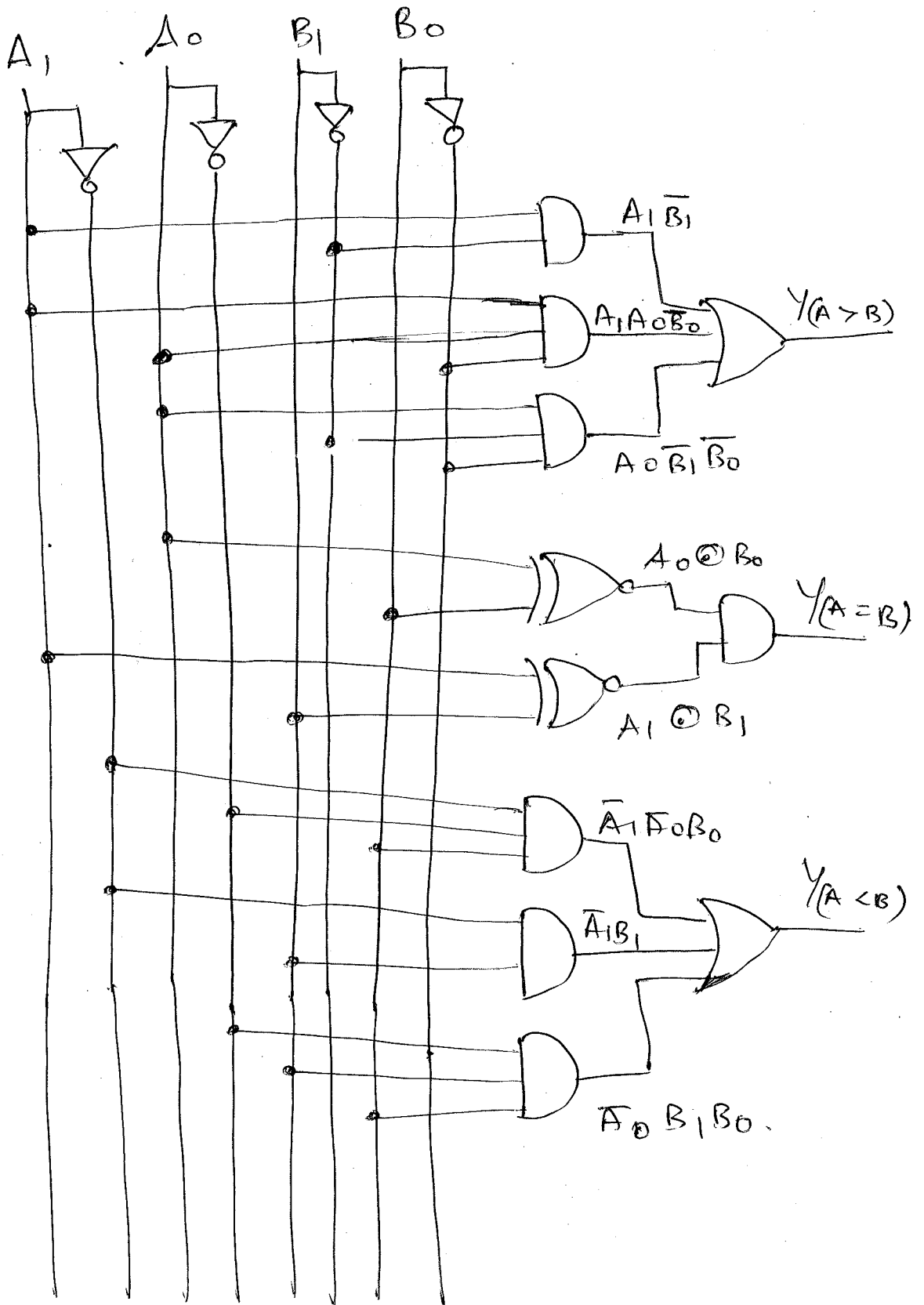
$$Y_{A=B} = (A_0 \odot B_0) (A_1 \odot B_1)$$



$$Y_{A < B} = \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0$$

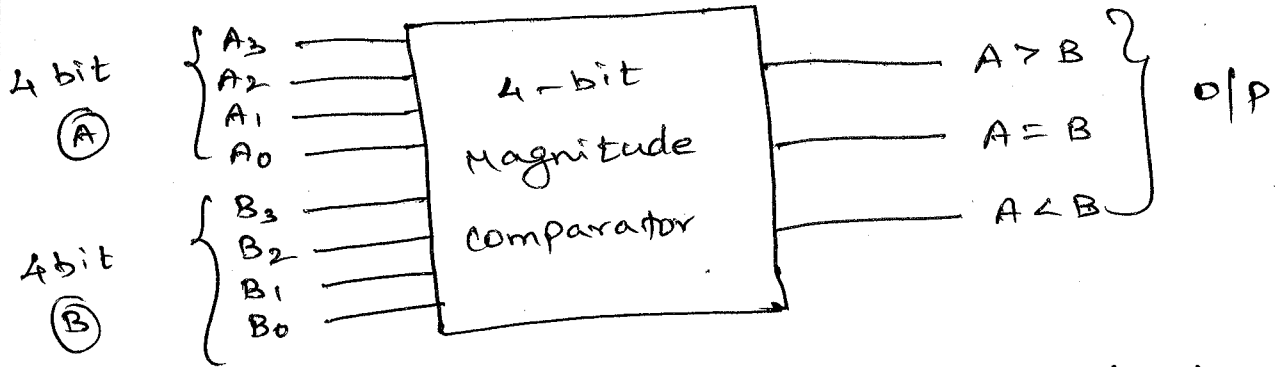
logic diagram

inputs



# 4-bit Magnitude Comparator

A magnitude comparator is a combinational ckt. that compare two numbers A & B. outcome of comparison are  $A > B$ ,  $A = B$  or  $A < B$ .



1)  $A = B$ : The two numbers are equal if all pairs of significant digits are equal:  $A_3 = B_3$ ,  $A_2 = B_2$ ,  $A_1 = B_1$ ,  $A_0 = B_0$ . The equality of each pair either '1' or '0' and the equality of each pair of bits can be expressed logically with an exclusive NOR function as  $x_i = A_i B_i + A_i' B_i'$  where  $i = 0, 1, 2, 3$

$$A = \overline{A_3 \oplus B_3} \cdot \overline{A_2 \oplus B_2} \cdot \overline{A_1 \oplus B_1} \cdot \overline{A_0 \oplus B_0}$$

$$B = \overline{B_3 \oplus A_3} \cdot \overline{B_2 \oplus A_2} \cdot \overline{B_1 \oplus A_1} \cdot \overline{B_0 \oplus A_0}$$

equal means both  $(A_i, B_i)$  should be '1' or '0' then  $A \oplus B = 1$

$$x_3 = \overline{A_3 \oplus B_3} = 1$$

$\therefore$  XNOR is used.

$$x_2 = \overline{A_2 \oplus B_2} = 1$$

$$x_1 = \overline{A_1 \oplus B_1} = 1$$

$$x_0 = \overline{A_0 \oplus B_0} = 1$$

$$A = B \Rightarrow \underbrace{x_3 x_2 x_1 x_0}_{=1}$$

AND gate

2)  $A > B$

$$A_3 > B_3 \Rightarrow A_3 B_3'$$

$$A_2 > B_2 \Rightarrow x_3 A_2 B_2'$$

$$A_1 > B_1 \Rightarrow x_3 x_2 A_1 B_1'$$

$$A_0 > B_0 \Rightarrow x_3 x_2 x_1 A_0 B_0'$$

If any one is high then  $\therefore$  OR operation,  $A > B$

Starting from MSB, check the magnitude of pairs. If 2 digit of pair are equal, we compare the next lower significant pair of digits. The comparison continues until pair of unequal digits is reached.  $A > B \rightarrow A=1, B=0$   
 $A < B \rightarrow A=0, B=1$

$$A > B = A_3 B_3' + x_3 A_2 B_2' + x_3 x_2 A_1 B_1' + x_3 x_2 x_1 A_0 B_0'$$

(10)

3) A < B

$$A_3 < B_3 \rightarrow A_3' B_3$$

$$A_2 < B_2 \rightarrow A_2' B_2$$

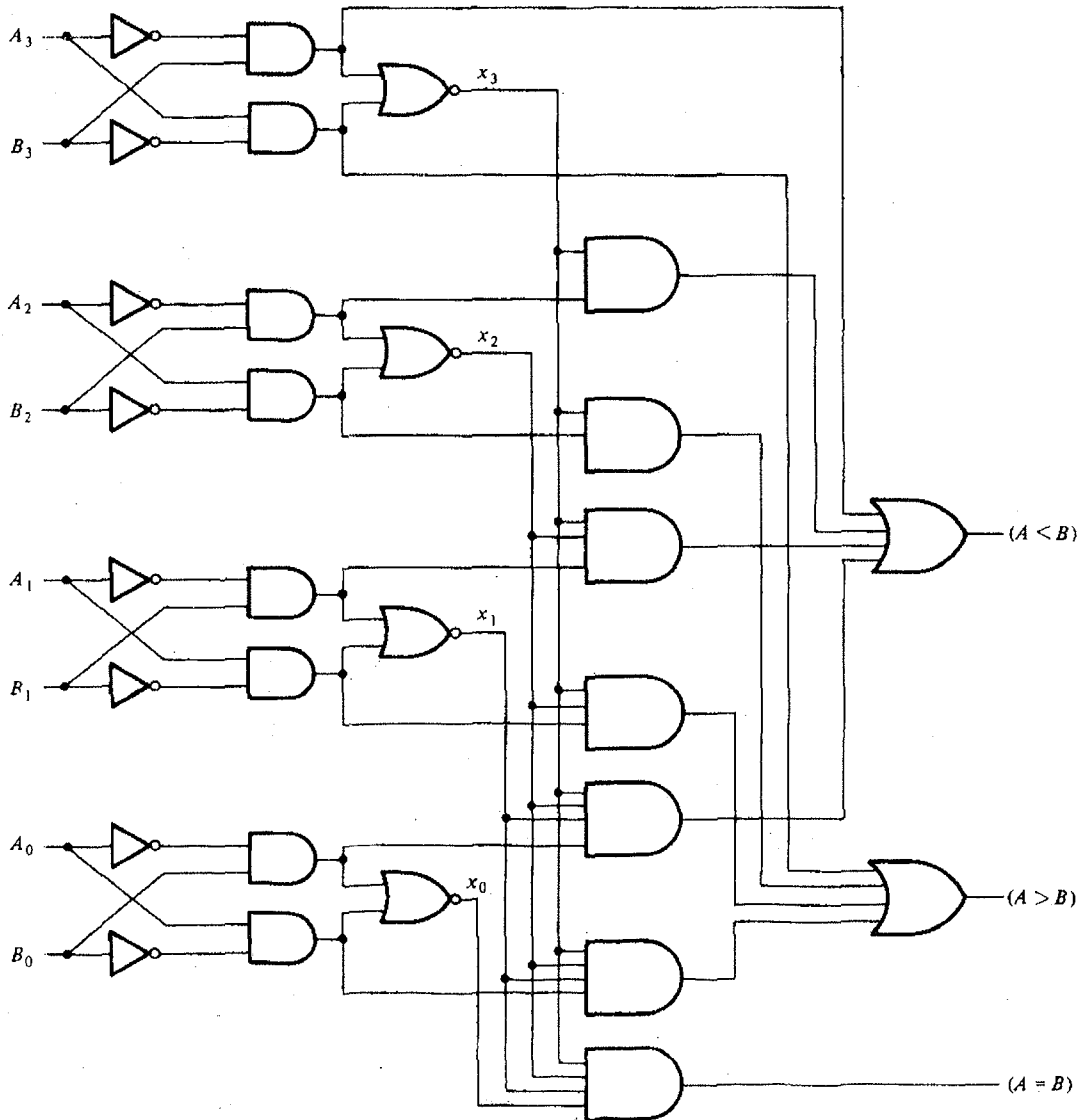
$$A_1 < B_1 \rightarrow A_1' B_1$$

$$A_0 < B_0 \rightarrow A_0' B_0$$

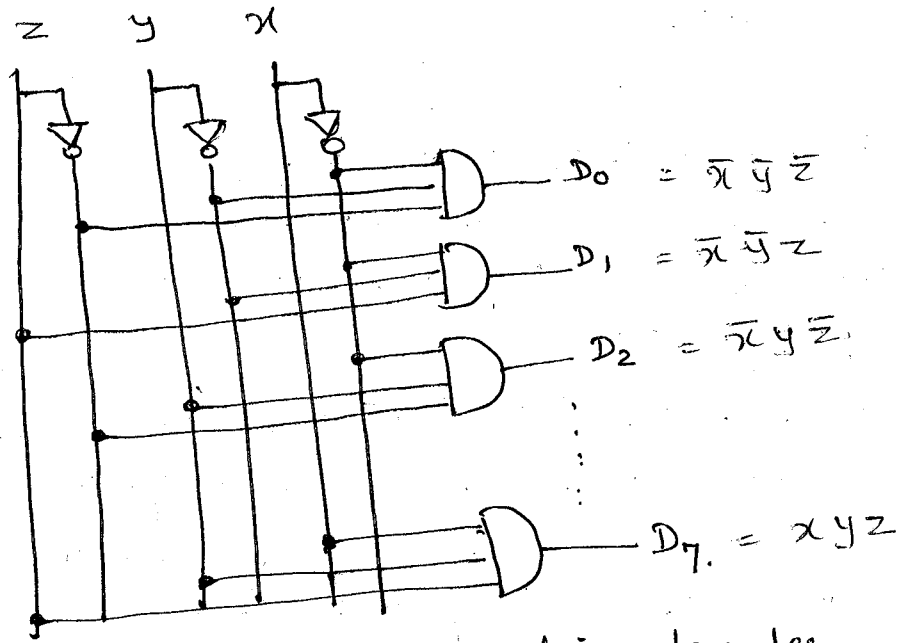
If any one is high, then  $A < B$

∴ OR operation

$$A < B = A_3' B_3 + x_3 A_2' B_2 + x_3 x_2 A_1' B_1 + x_3 x_2 x_1 A_0' B_0$$



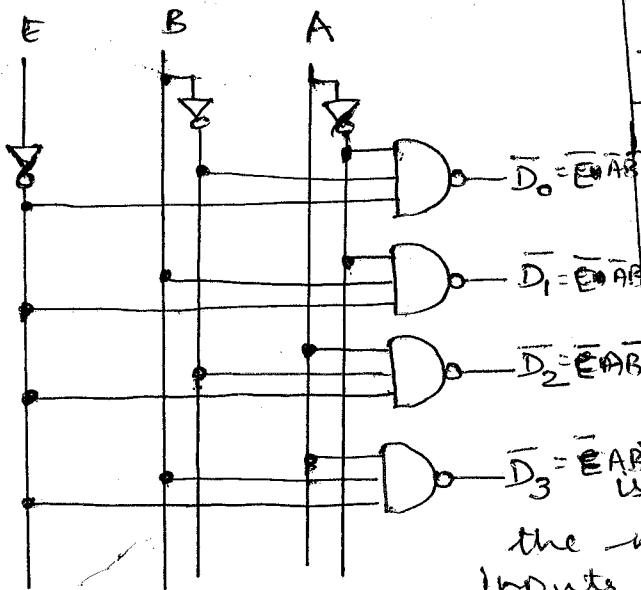




3 : 8 line decoder.

CONSTRUCT DECODER WITH NAND.

- \* AND with Inverter output.
- \* Decoder includes one or more enable input to control the circuit operation.
- \* Decoder is enabled when  $E = 0$
- \* Only one output can be equal to 0 at any given time all other outputs are equal to 1.



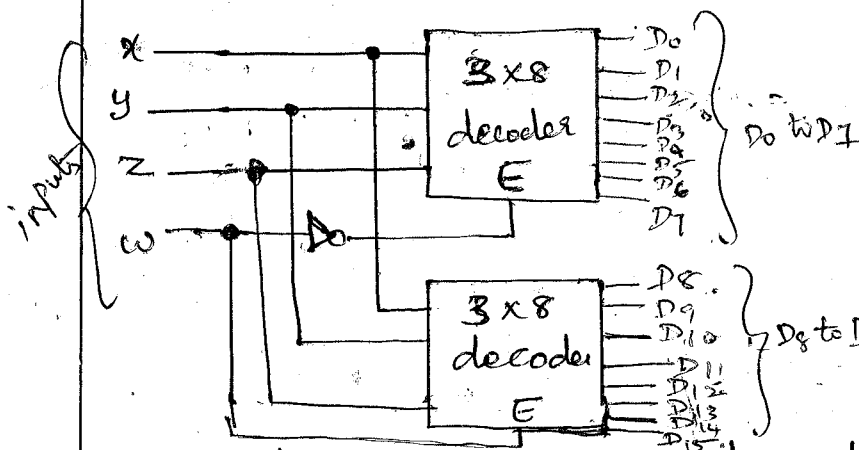
Input			output			
E	A	B	$\bar{D}_0$	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

\* The outputs whose value is equal to 0 represents the minterm selected by inputs A and B. (active low)

\* circuit disabled when  $E = 1$ , regardless of the values of other two inputs.

## A 16 Decoder constructed with two

3x8 decoder.

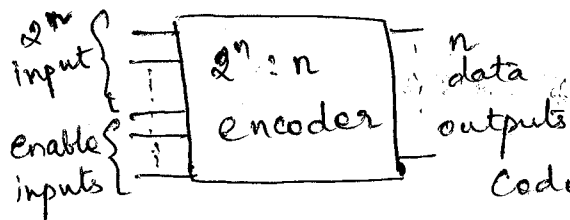


\* When  $w=0$ , top decoder is enabled and the other disabled.

\* The bottom decoder output are all 0's and top 8 output generate minterms 0000 to 0111.

\* When  $w=1$ , bottom decoder only generate minterms 1000 to 1111.

## ENCODER.



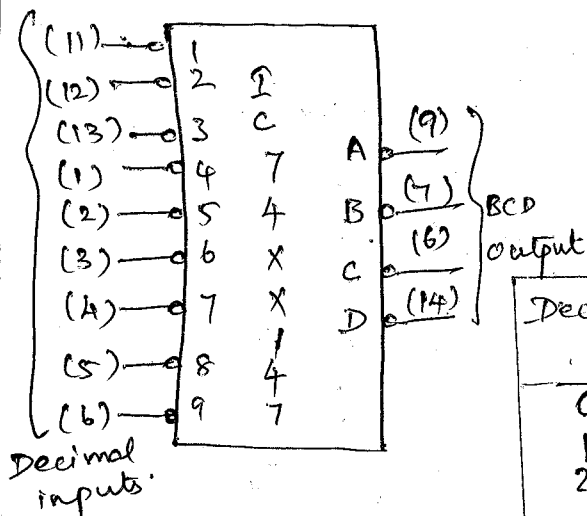
\*  $2^n$  input lines and n output lines.

\* output lines generate binary code corresponding to input value.

## DECIMAL TO BCD ENCODER.

Decimal  $\rightarrow$  10 input lines.

BCD  $\rightarrow$  4 output lines.



\* active low  $\rightarrow$  input and output

\* No input line for decimal zero.

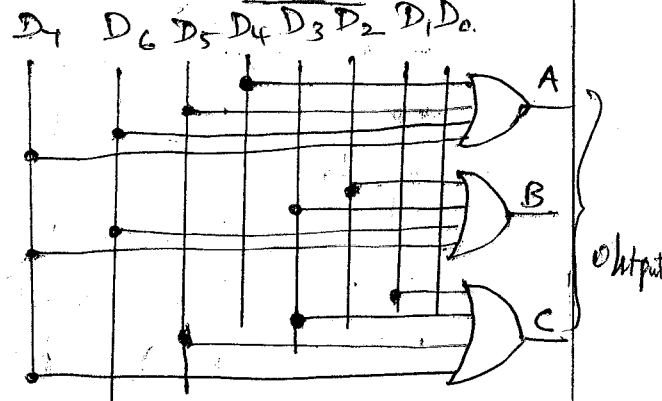
\* When this condition occurs all input lines are zero.

Decimal	Inputs									Output			
	1	2	3	4	5	6	7	8	9	D	C	B	A
0	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1	0	0
2	X	0	1	1	1	1	1	1	1	1	1	0	1
3	X	X	0	1	1	1	1	1	1	1	1	0	0
4	X	X	X	0	1	1	1	1	1	1	1	0	1
5	X	X	X	X	0	1	1	1	1	1	1	0	0
6	X	X	X	X	X	0	1	1	1	1	1	0	0
7	X	X	X	X	X	X	0	1	1	1	1	0	0
8	X	X	X	X	X	X	X	0	1	1	1	0	1
9	X	X	X	X	X	X	X	X	0	1	1	0	0

### Octal to binary encoder.

- \* eight inputs, three outputs
- \* Only one input has the value 1 at any given time

Input.								Output		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1



- \* When all inputs are zero, the outputs are zero.
- \* Zero output can be generated when D<sub>0</sub> = 1.

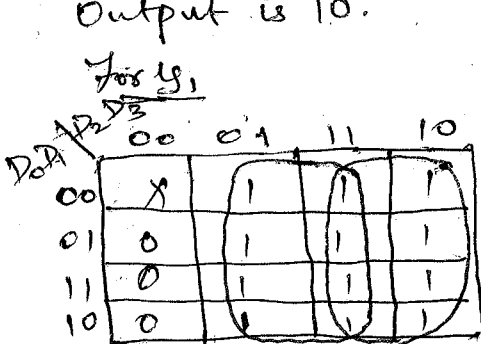
### PRIORITY ENCODER.

- \* Priority function
- \* If 2 or more inputs are equal to 1 at the same time, the input having highest priority.

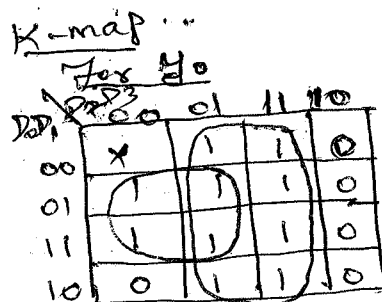
Inputs.				Output		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Y <sub>1</sub>	Y <sub>0</sub>	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

- \* V → Valid output indicators
- \* D<sub>3</sub> has the highest priority and D<sub>0</sub> has lowest priority.
- \* When D<sub>3</sub> input is high, regardless of other inputs output is 11.

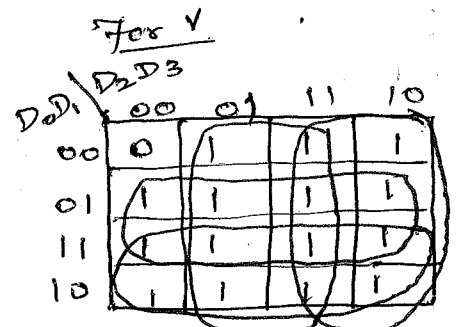
- \* D<sub>2</sub> has next priority.
- \* D<sub>3</sub> = 0, D<sub>2</sub> = 1 regardless of lower priority input, output is 10.



$Y_1 = D_2 + D_3$

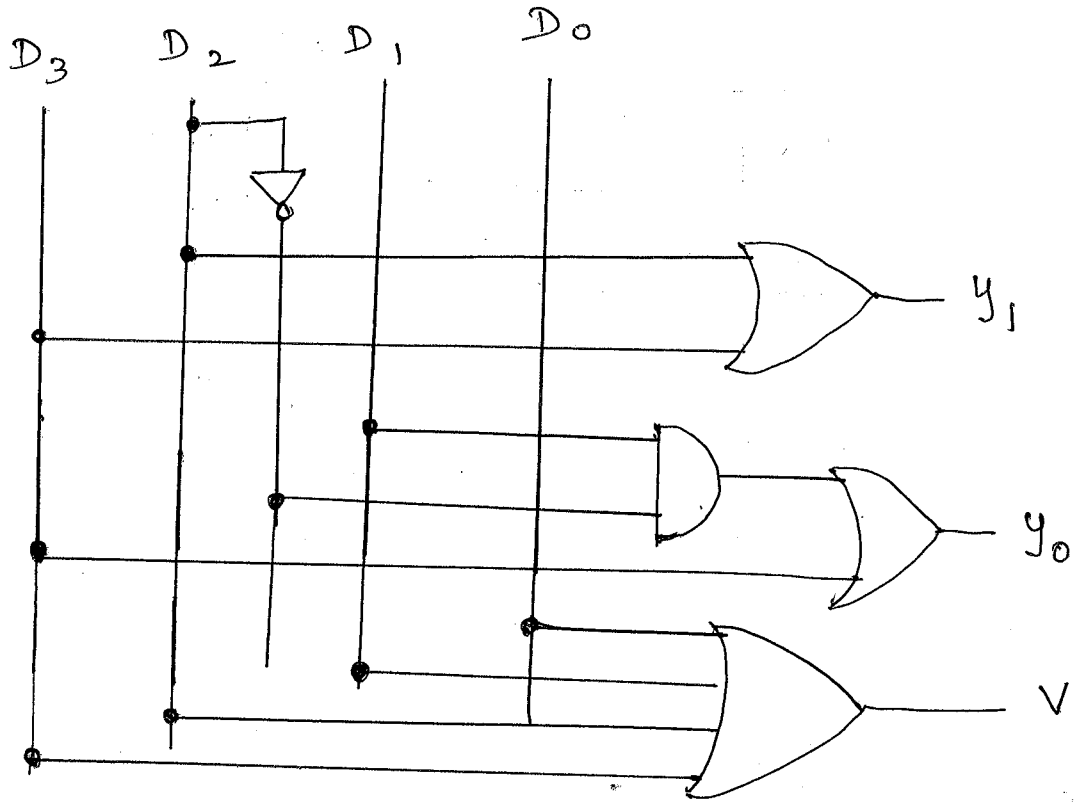


$Y_0 = D_1 \bar{D}_2 + D_3$



$V = D_0 + D_1 + D_2 + D_3$

logic diagram



Parity generator & checker

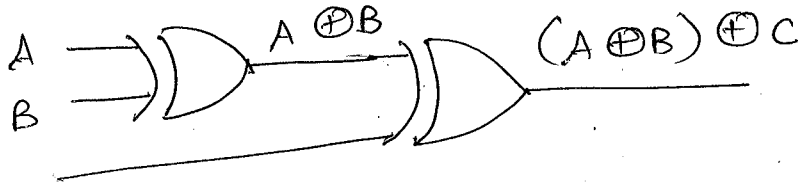
Parity generator

3 bit message			odd Parity bit	even parity bit
A	B	C		
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

Even parity generator.

A \ B <sup>c</sup>	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\begin{aligned}
 &= \bar{A}\bar{B}c + \bar{A}B\bar{c} + A\bar{B}\bar{c} + ABC \\
 &= \bar{A}(\bar{B}c + B\bar{c}) + A(\bar{B}\bar{c} + BC) \\
 &= \bar{A}(B \oplus c) + A(\overline{B \oplus c}) \\
 &= A \oplus B \oplus c
 \end{aligned}$$



Even parity checker

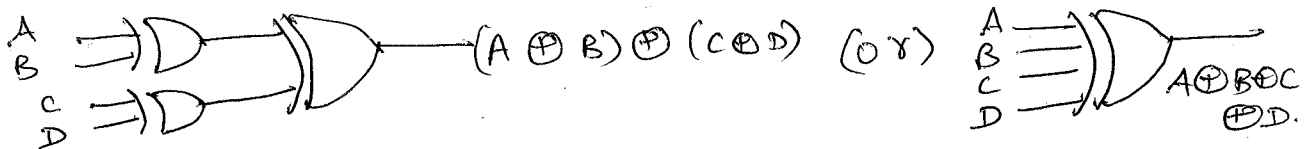
\* If zero accepts

Decimal	A bit received				parity error check
	A	B	C	D	
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	0

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$$\begin{aligned}
 &= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} \\
 &+ \bar{A}BCD + AB\bar{C}D + ABC\bar{D} \\
 &+ A\bar{B}\bar{C}D + A\bar{B}C\bar{D} \\
 &= \bar{A}\bar{B}(\bar{C}D + C\bar{D}) + \\
 &\bar{A}B(\bar{C}D + CD) + \\
 &AB(\bar{C}D + C\bar{D}) + \\
 &A\bar{B}(\bar{C}D + CD)
 \end{aligned}$$

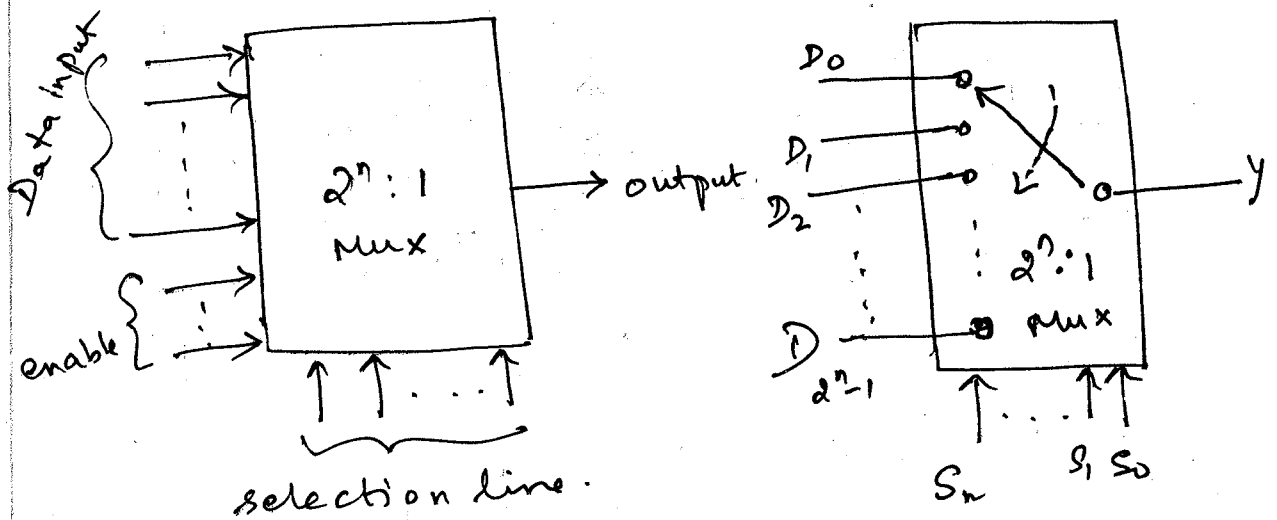
$$\begin{aligned}
 &= \bar{A}\bar{B}[C \oplus D] + \bar{A}B[\overline{C \oplus D}] + AB[C \oplus D] + A\bar{B}[\overline{C \oplus D}] \\
 &= [C \oplus D][\bar{A}\bar{B} + AB] + [\overline{C \oplus D}][\bar{A}B + A\bar{B}] \\
 &= [C \oplus D][\overline{A \oplus B}] + [\overline{C \oplus D}][A \oplus B] = [C \oplus D] \oplus [A \oplus B]
 \end{aligned}$$



## Multiplexer

It selects binary information from many input lines and directs it to a single output line.  $2^n:1$  selection lines.   
 ↑ input ← output

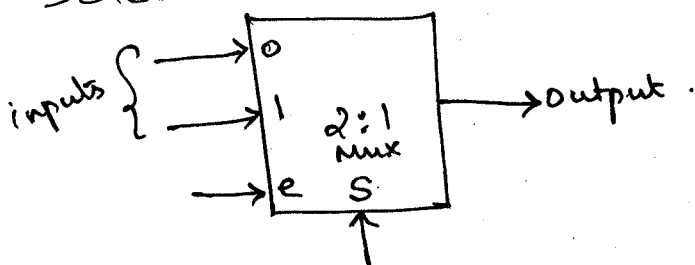
The selection of particular input lines is controlled by a set of selection lines. enable or strobe is used for cascading.



It is otherwise called data selector and is used in D/A converter, data acquisition system.

### 2 to 1 line Multiplexer

It has 2 inputs and one output. Since binary input is two, ~~enable~~ <sup>need</sup> is only one. Selection line.



Truth table

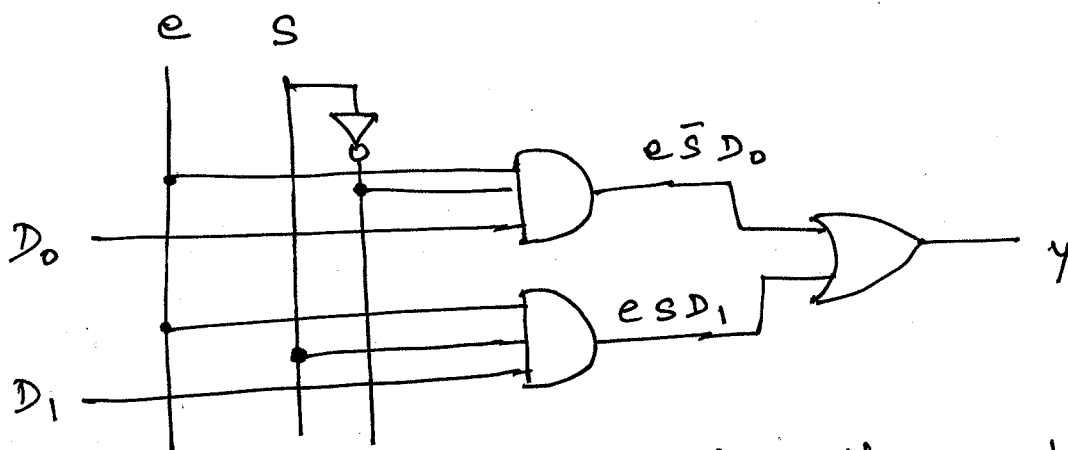
e	s	Y
1	0	D <sub>0</sub>
	1	D <sub>1</sub>
0	x	0

$e\bar{s}D_0$

$e s D_1$

$Y = e\bar{s}D_0 + e s D_1$

logic diagram .



Case i)  $s = 0, e = 1, D_0 = 1$ , then the output is  $D_0$ .

otherwise output is '0'.

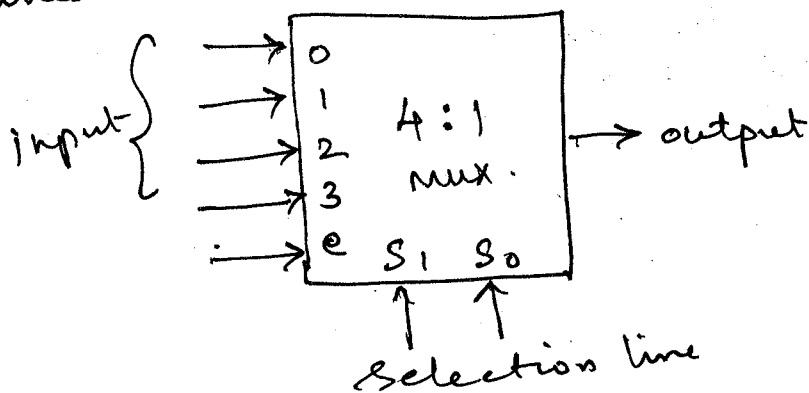
Case ii)  $s = 1, e = 1, D_1 = 1$ , then output is  $D_1$ ,

otherwise output is '0'.

4 to 1 line multiplexer .

It has ~~four~~ inputs and one output .

since 4 inputs ( $2^2$ ) it needs two selection lines .



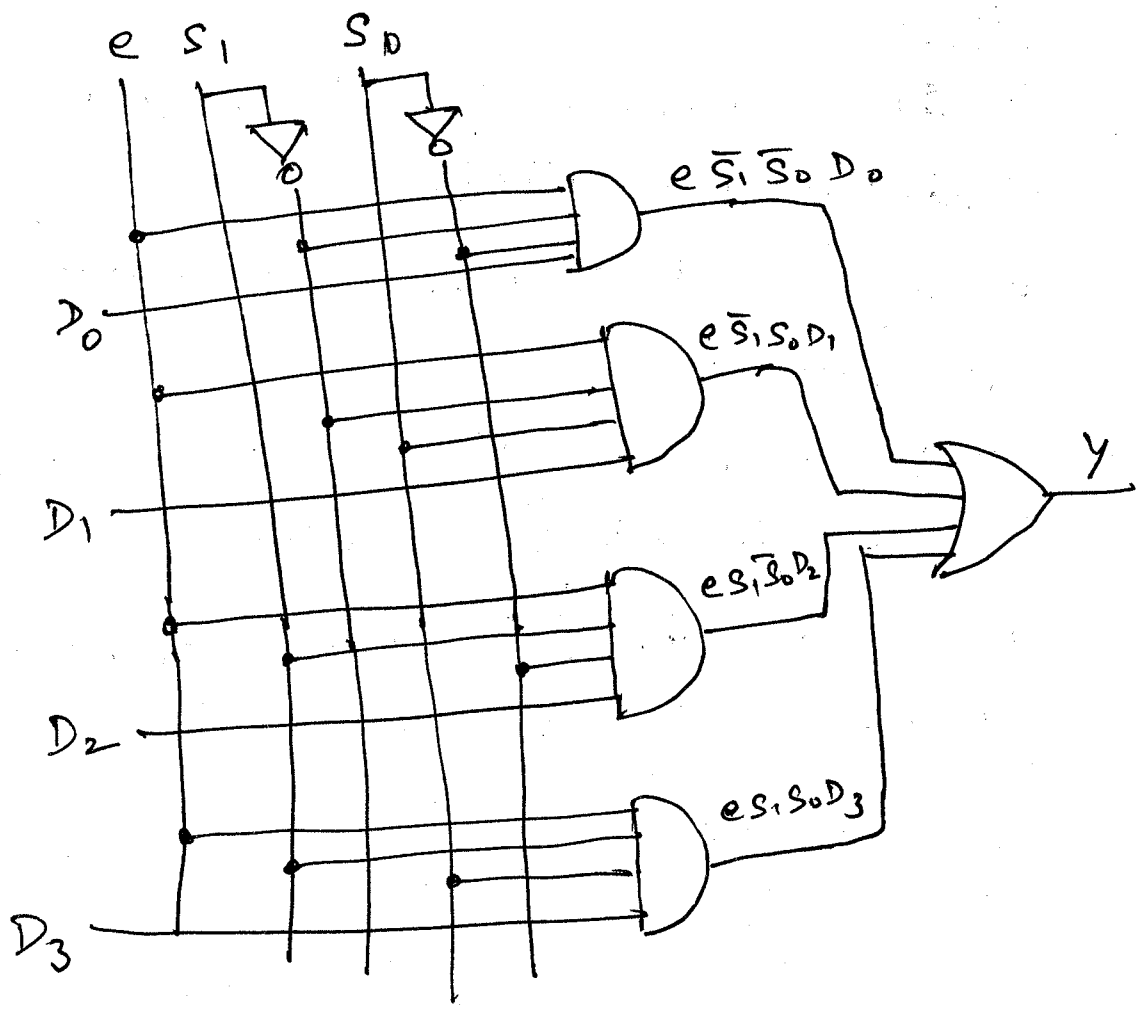
Truth table

e	s <sub>1</sub>	s <sub>0</sub>	y
1	0	0	D <sub>0</sub>
1	0	1	D <sub>1</sub>
1	1	0	D <sub>2</sub>
1	1	1	D <sub>3</sub>
0	x	x	0

$e \bar{s}_1 \bar{s}_0 D_0$   
 $e \bar{s}_1 s_0 D_1$   
 $e s_1 \bar{s}_0 D_2$   
 $e s_1 s_0 D_3$

$Y = e \bar{s}_1 \bar{s}_0 D_0 + e \bar{s}_1 s_0 D_1 + e s_1 \bar{s}_0 D_2 + e s_1 s_0 D_3$

Logic diagram



If enable  $e = 1$

Case i)  $S_1 = 0, S_0 = 0, D_0 = 1$  then output is  $D_0$  otherwise output is '0'

Case ii)  $S_1 = 0, S_0 = 1, D_1 = 1$  then output is  $D_1$ , otherwise output is '0'

Case iii)  $S_1 = 1, S_0 = 0, D_2 = 1$ , then output is  $D_2$ , otherwise output is '0'.

Case iv)  $S_1 = 1, S_0 = 1, D_3 = 1$ , then output is  $D_3$ , otherwise output is '0'.

### Implementation of Multiplexer.

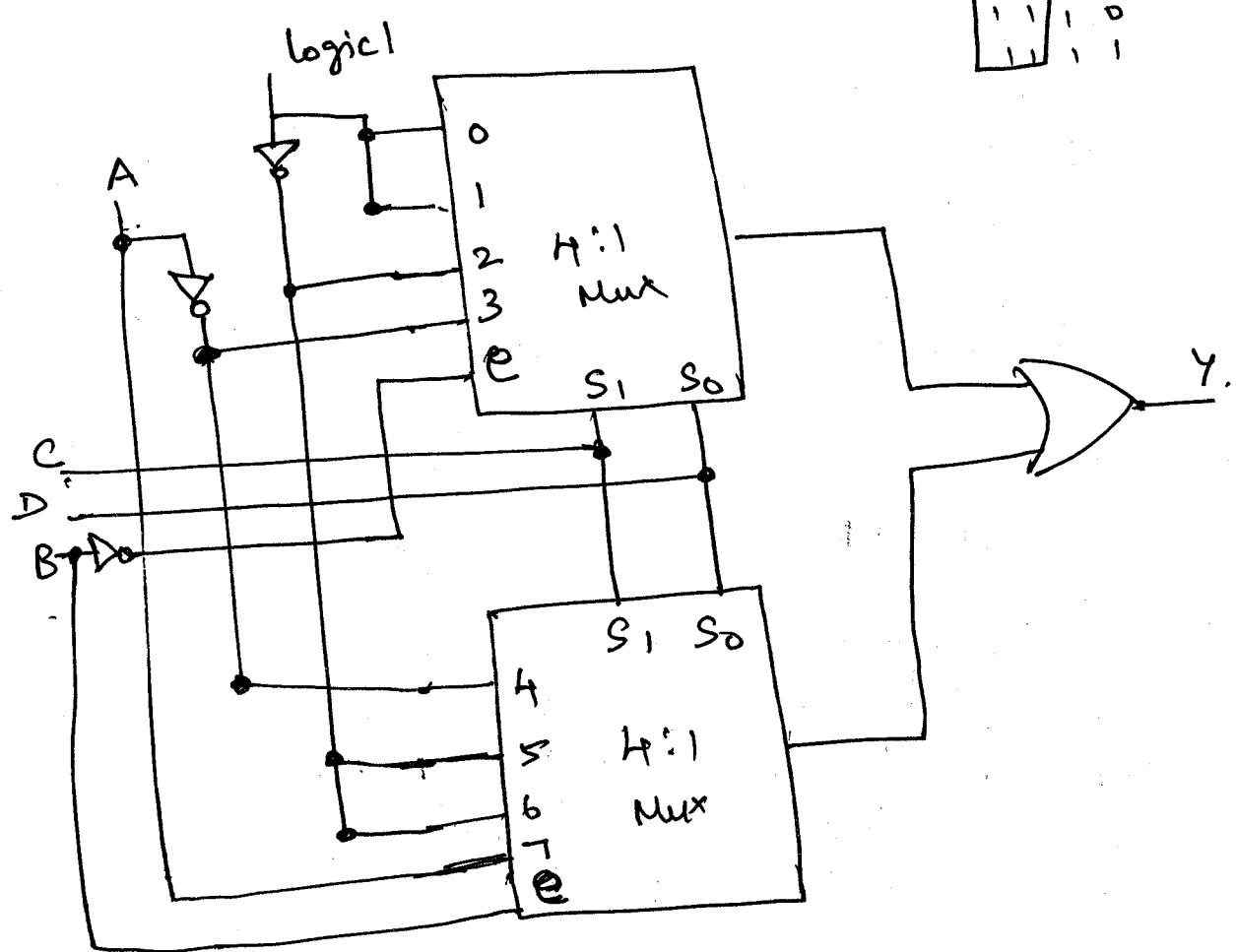
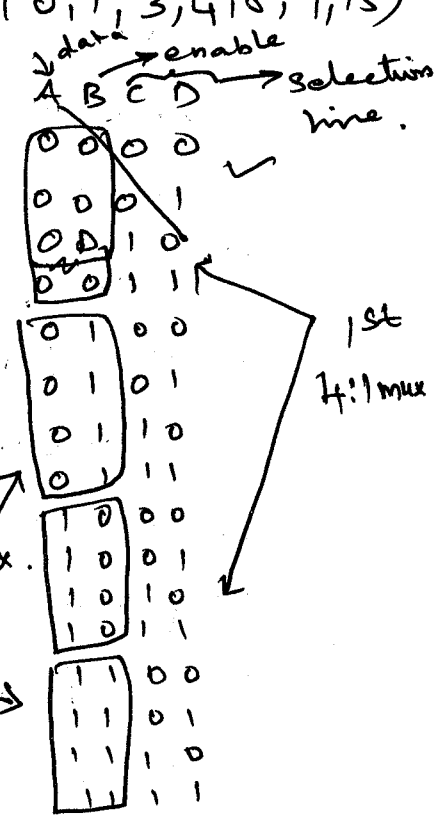
steps:

1. If 2 minterms in column are not circled '0' is applied.
2. If 2 minterms in column are circled the value is '1'.
3. Minterm in 1<sup>st</sup> row is circled, represent the Complemented variable.
4. Minterm in 2<sup>nd</sup> row is circled and minterm represent as variable.

Implement the given function using

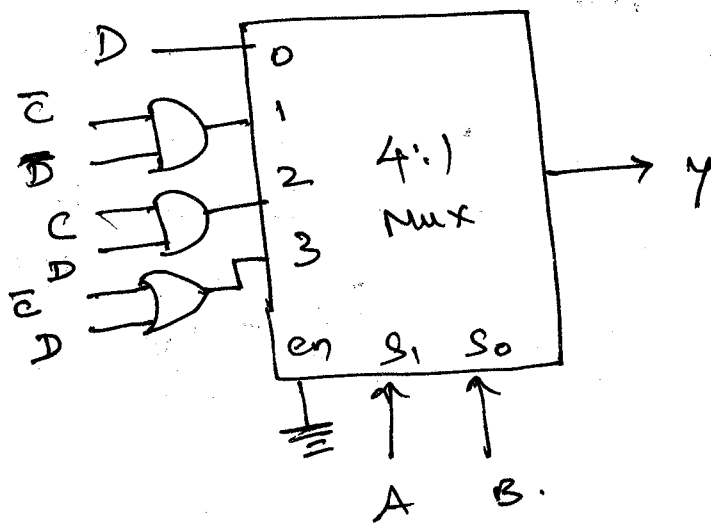
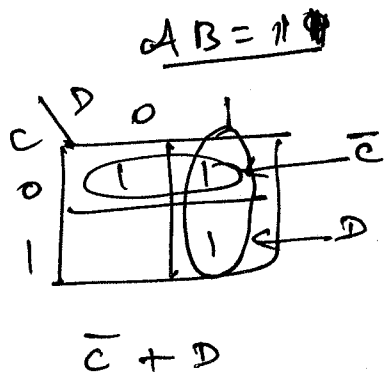
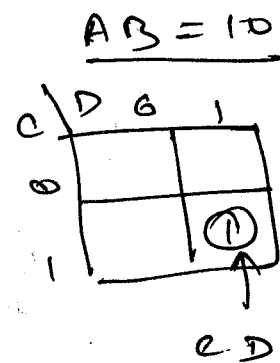
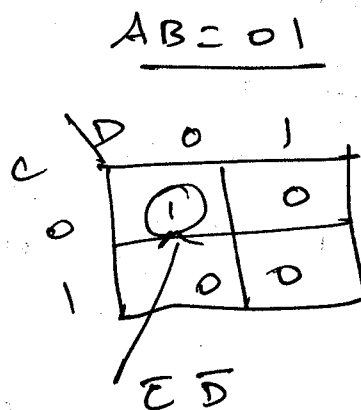
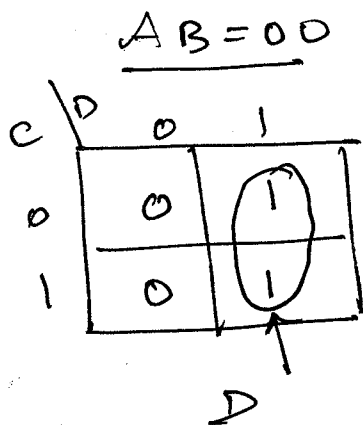
A:1 Mux.  $F(A, B, C, D) = \sum m(0, 1, 3, 4, 8, 9, 15)$

	000	001	010	011	100	101	110	111
	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
$\bar{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	1	0	$\bar{A}$	$\bar{A}$	0	0	A



Implement the following boolean function with a 4 to 1 mux and external gates. Connect inputs A and B to selection line. The input requirement for 4 data lines will be function of variable C and D. These values are obtained by expressing F as function of C & D for each of 4 cases when AB = 00, 01, 10, 11. These functions may have to be implemented with external gates.

$$F(A, B, C, D) = \sum m(1, 3, 4, 11, 12, 13, 15)$$



Implement using Multiplexer (IC 74150)

1.  $F(x, y, z) = \sum(1, 3, 5, 6)$

Data →

	x	yz	F
0	0	00	0
1	0	01	1
2	0	10	1
3	0	11	0
4	1	00	0
5	1	01	0
6	1	10	1
7	1	11	1

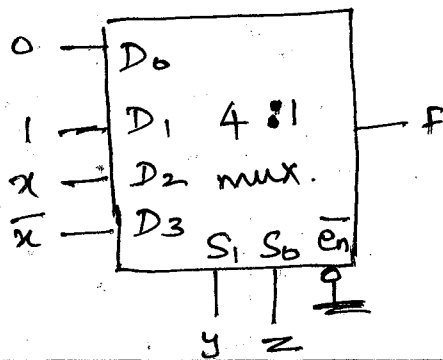
	00	01	10	11
$\bar{x}$	$D_0$	$D_1$	$D_2$	$D_3$
	0	①	2	③
x	4	⑤	⑥	7
	0	1	x	$\bar{x}$

\* 2 minterm in column are not circled '0' is applied.

\* 2 minterm are circled value = 1

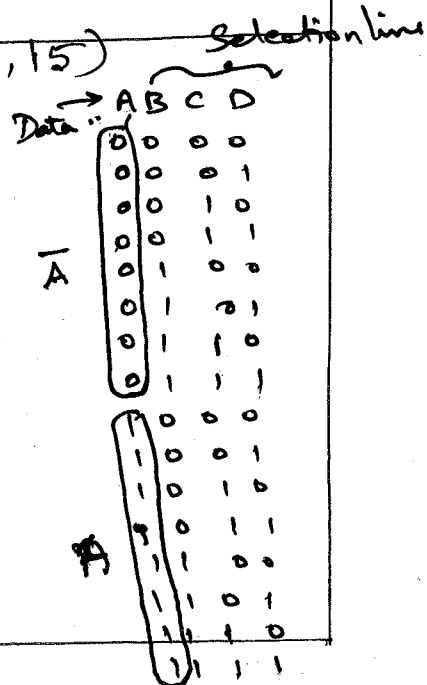
\* Minterm in 2<sup>nd</sup> row is circled and minterm is the variable.

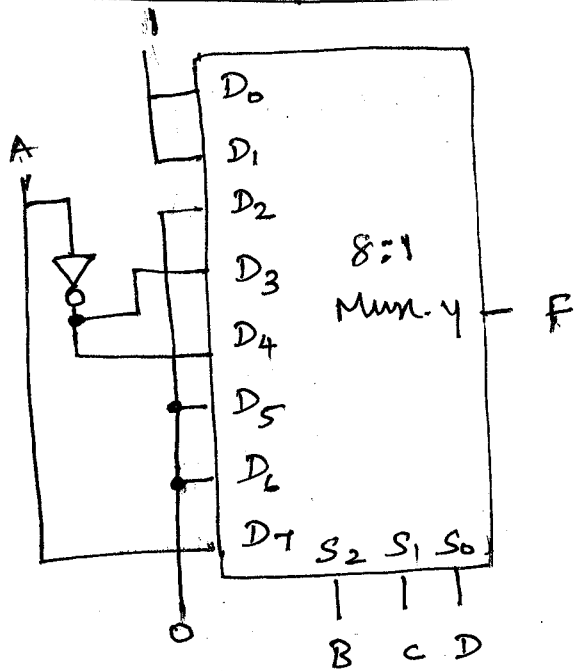
\* Minterm in 1<sup>st</sup> row is circled, represent the complemented variable.



2.  $F(A, B, C, D) = \sum m(0, 1, 3, 4, 8, 9, 15)$

	$D_0$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$
$\bar{A}$	①	②	3	④	5	6	7	
A	⑧	⑨	10	11	12	13	14	⑮
	1	1	0	$\bar{A}$	$\bar{A}$	0	0	A

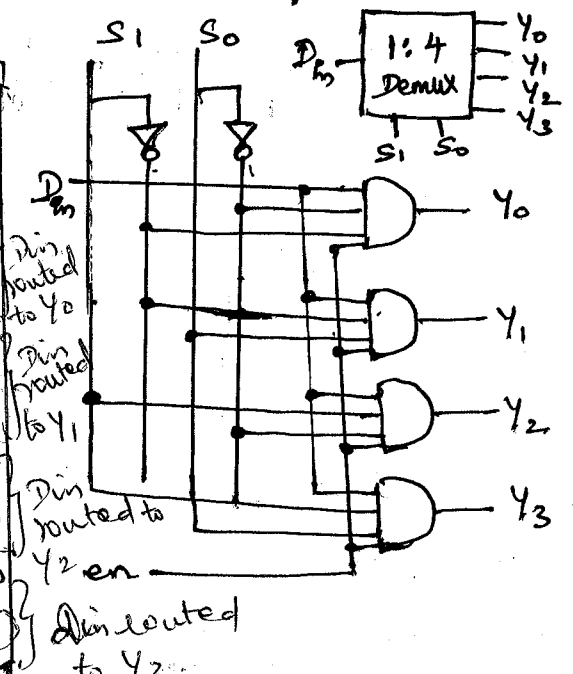




DEMULTIPLEXER.

\* receives information on a single line and transmits this information on one of  $2^n$  possible output lines.

en	S <sub>1</sub>	S <sub>0</sub>	D <sub>in</sub>	Y <sub>0</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
0	x	x	x	0	0	0	0
1	0	0	0	0	0	0	0
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	0	0	1	0	0	0
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	0	1	0	0	0	0
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	1	0	0	1	0	0
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	1	1	0	0	0	1
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	1	1	0	0	0	0
en S <sub>1</sub> S <sub>0</sub> D <sub>in</sub>	1	1	1	1	0	0	0

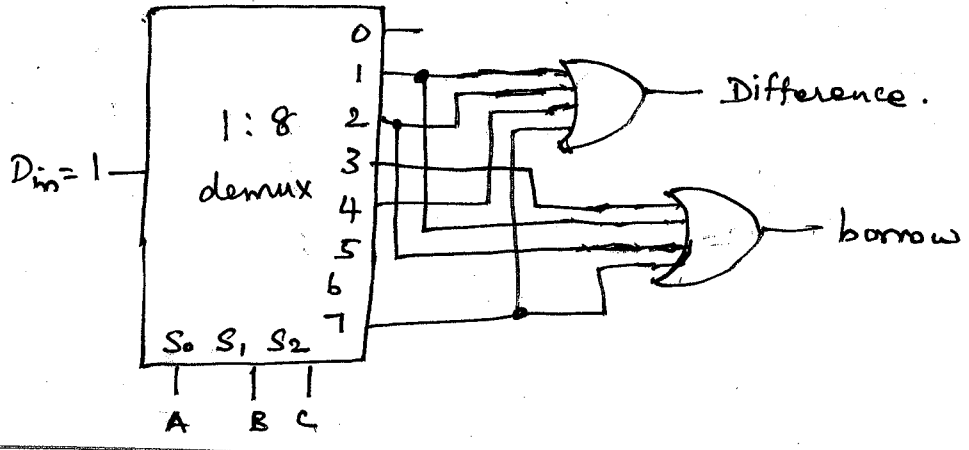


Implement full subtractor using demultiplexer. (IC 74154)

A	B	B <sub>in</sub>	D	B <sub>out</sub>
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0

A	B	B <sub>in</sub>	D	B <sub>out</sub>
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$D = \sum m(1, 2, 4, 7)$  ,  $B = \sum m(1, 2, 3, 7)$ .

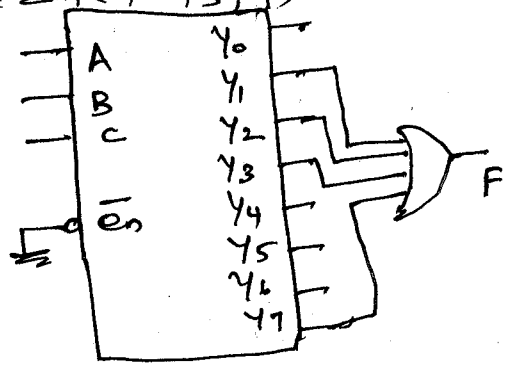


Realization of multiple output function using binary decoder.

For active high output.

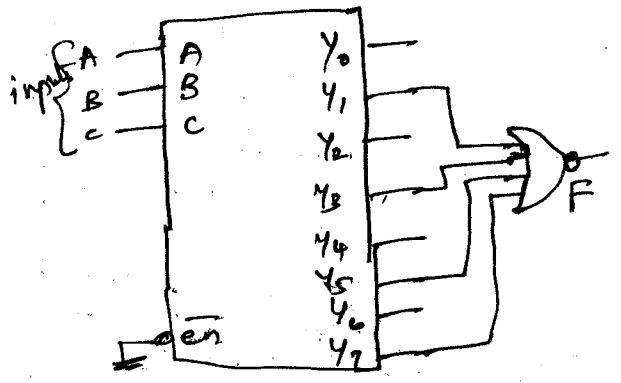
SOP function implementation

$f = \sum m(1, 2, 3, 7)$



POS function implementation

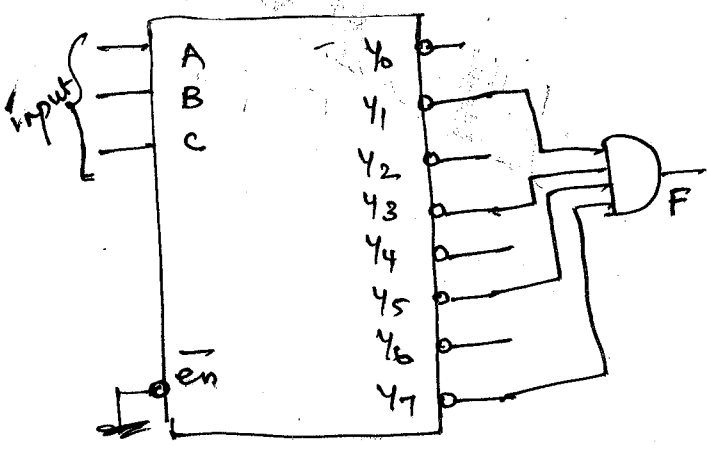
$f = \prod m(1, 3, 5, 7)$



For Active low output

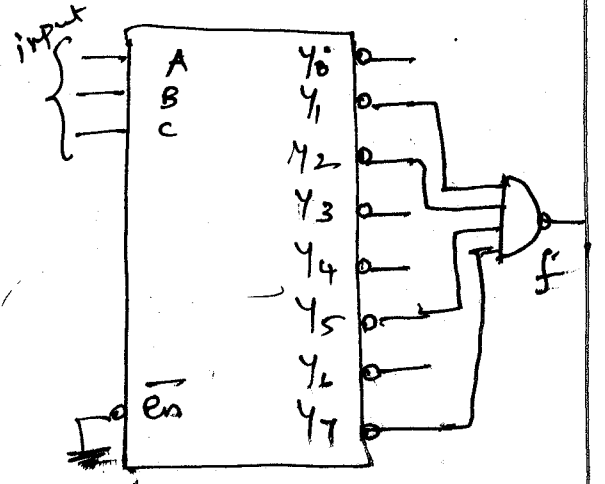
POS function implementation

$f = \prod m(1, 3, 5, 7)$



SOP function implementation

$f = \sum m(1, 2, 5, 7)$



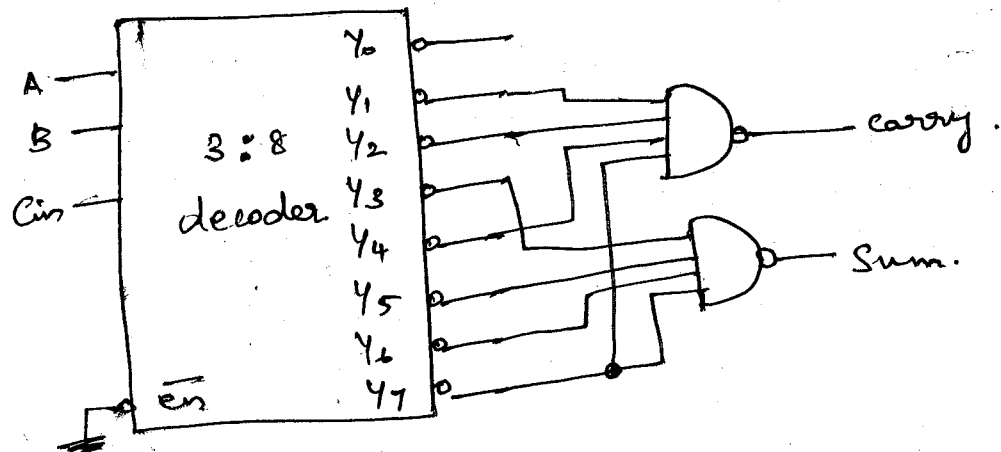
Design full adder use 3:8 decoder.

Input			Carry	Sum
A	B	Cin		
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$\text{sum} = x'y'z + x'y'z' + xy'z' + xyz$$

$$\text{Sum} = \sum m(1, 2, 4, 7)$$

$$\begin{aligned} \text{Carry} &= xy + xz + yz \\ &= \sum m(3, 5, 6, 7) \end{aligned}$$



### Multiplexer Vs Decoder.

- \* Multiplexer has several data-input lines and a single output line. The selection of a particular input line is controlled by set of selection lines.
- \* The data of selected input line is routed to output line.
- \* Decoder has 'n' input lines and  $2^n$  output lines. Each of line represent one minterm.
- \* Decoder activates one of the output line depending on the input combination.